NCERT Class 10 Solutions: Real Numbers (Chapter 1) Exercise 1.1 – Part 2

Q-3 An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

For the above problem , the maximum number of columns would be the HCF of 616 and 32

We can find the HCF of 616 and 32 by using Euclid Division algorithm.

Therefore,

$$616 = 19 \times 32 + 8$$

Since remainder $8 \neq 0$, we apply the division lemma to 32 and 8 to obtain

$$32 = 4 \times 8 + 0$$

Therefore $HCF(616, 32) = HCF(32, 8) = 8$

Therefore, they can march in 8 columns each.

Q-4 Use Euclid's division lemma to show that the square of any positive integer is either of form 3*m* or 3*m* + 1 for some integer *m*.

[**Hint**: Let *x* be any positive integer then it is of the form $3q, 3q + 1 \text{or} 3q + 2$. Now square each of these and show that they can be rewritten in the form $3m \text{or} 3m + 1$.]

Solution:

Euclid's Division Lemma

QUESTION

Show that the square of any positive integer is of the form 4p or 4p+1 for some integer "p"

SOLUTION

Let ' m' be the positive integer

Case 1

If "m" is Even

when m = 2q

$m^2 = (2q)^2$

$m^2 = 4q^2$

$m^2 = 4q(q+0)$

$m^2 = 4p$ $\{\because p = q(q+0)\}$

Case 2

If "m" is Odd

when m= 2q + 1

$m^2 = (2q + 1)^2$

$m^2 = 4q^2 + 4q + 1$

$m^2 = 4q(q+1) + 1$

$m^2 = 4p + 1$ $\{\because p = q(q+1)\}$

∴The square of any positive integer is of the form 4p or 4p + 1 for some integer "p"

According to Euclid algorithm

We have $a = bq + r$ (Equation 1)

And substituting $b = 3$ in equation 1, we get

$a = 3q + r, \{r \leqslant 0 < 3\}$

$r = 0, 1, 2$

When $r = 0, a = 3q$ or $a^2 = 9q^2$ (Equation A)

When $r = 1, a = 3q + 1$ or $a^2 = 9q^2 + 1 + 6q$ (Equation B)

When $r = 2, a = 3q + 2$ or $a^2 = 9q^2 + 4 + 12q$ (Equation C)

We can be rewrite equation A as $a^2 = 3\left(3q^2\right)$ say 3m

Where, $m = 3q^2$

Also equation B can be written as $a^2 = 3\left(3q^2 + 2q\right) + 1$ or $a^2 = 3m + 1$

Where, $m = 3q^2 + 2q$

Also equation C can be written as $a^2 = 3\left(3q^2 + 4q + 1\right) + 1$ or $a^2 = 3m + 1$

Where, $m = 3q^2 + 4q + 1$

Hence, the square of any positive integer is either of the form $3m \text{or} 3m+1$ for some integer m.

Q-5 Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m, 9m+1 \text{or} 9m+8$.

Solution:

We know that by using Euclid's Division Algorithm, $a = bq + r$

Substituting $b = 9$, we get

$a = 9q + r$ Where, $\{r \leqslant 0 < 9\}$

$r = 0, 1, 2, 3, 4, 5, 6, 7, 8$

When $r = 0, a = 9q$

$a^3 = 729q^3 = 9\left(81q^3\right) = 9m$, where $m = 81q^3$

When $r = 1, a = 9q + 1$

$a^3 = 729q^3 + 243q^2 + 27q + 1$

$= 9\left(81q^3 + 27q^2 + 3q\right) + 1 = 9m + 1$, where $m = 81q^3 + 27q^2 + 3q$

When $r = 4, a = 9q + 4$

$a^3 = 729q^3 + 972q^2 + 432q + 64$

$= 9\left(81q^3 + 108q^2 + 48q + 7\right) + 1$

Continuing the process till $r = 8 \text{and} a = 9q + 8$, we get

$a^3 = 729q^3 + 1944q^2 + 1728q + 512$

$= 9\left(81q^3 + 216q^2 + 192q + 56\right) + 8$

$= 9m + 8$

Where, $m = 81q^3 + 216q^2 + 192q + 56$

Hence, it is proved that any positive integer is either of the form $9m, 9m+1 \text{or} 9m+8$