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NCERT Class 12- Mathematics: Chapter – 9 Differential Equations Part 9

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Long Answer (L. A.)

Question 25:

Solve: $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

Answer:

Given differential equation is

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$\Rightarrow y + x \frac{d}{dx} y + y = x(\sin x + \log x) \quad y + x \frac{dy}{dx} + y = x(\sin x + \log x)$$

$$\Rightarrow x \frac{dy}{dx} + 2y = x(\sin x + \log x)$$

Dividing both sides by x we get

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{2}{x}, Q = \sin x + \log x$$

$$\text{IF} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$$

The complete solution is

$$y \times \text{IF} = \int (\sin x + \log x) x^2 dx + C$$

The general solution is

$$y \cdot x^2 = \int (\sin x + \log x) x^2 dx + C$$

$$\Rightarrow y \cdot x^2 = \int (x^2 \sin x + x^2 \log x) dx + C$$

$$\Rightarrow y \cdot x^2 = \int x^2 \sin x dx + \int x^2 \log x dx + C$$

$$\Rightarrow y.x^2 = I_1 + I_2 + C \dots (i)$$

Now, $I_1 = \int x^2 \sin x dx$

$$= x^2 (\cos x) + \int 2x \cos x dx$$

$$= -x^2 \cos x + \left[2x (\sin x) - \int 2 \sin x dx \right]$$

$$I_1 = -x^2 \cos x + 2x \sin x + 2 \cos x \dots (ii)$$

And $I_2 = \int x^2 \log x dx$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} \dots (iii)$$

On substituting of value of I_1 and I_2 in Eq. (i), we get

$$y.x^2 = -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 + C$$

$$\therefore y = -\cos x + \frac{2 \sin x}{x} + \frac{2 \cos x}{x^2} + \frac{x}{3} \log x - \frac{x}{9} + Cx^{-2}$$

Question 26:

Find the general solution of $(1 + \tan y)(dx - dy) + 2xdy = 0$.

Answer:

Given differential equation is $(1 + \tan y)(dx - dy) + 2xdy = 0$

On dividing throughout by dy , we get

$$(1 + \tan y) \left(\frac{dx}{dy} - 1 \right) + 2x = 0$$

$$\Rightarrow (1 + \tan y) \frac{dx}{dy} - (1 + \tan y) + 2x = 0$$

$$\Rightarrow (1 + \tan y) \frac{dx}{dy} + 2x = (1 + \tan y)$$

Dividing both sides by $(1 + \tan y)$

$$\Rightarrow \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

Which is a linear differential equation

On comparing it with $\frac{dx}{dy} + Px = Q$ we get

$$P = \frac{2}{1 + \tan y}, Q = 1$$

$$IF = e^{\int dy \frac{2}{1 + \tan y}} e^{\int \frac{2 \cos y}{\cos y - \sin y} dy}$$

$$\begin{aligned}
 & e^{\int \frac{\cos y + \sin y + \cos y - \sin y}{\cos y - \sin y} dy} \\
 &= e^{\int \left(1 + \frac{\cos y - \sin y}{\cos y - \sin y}\right) dy} = e^{y + \log(\cos y + \sin y)} \\
 &= e^y \cdot (\cos y + \sin y) \left[\because e^{\log x} = x\right]
 \end{aligned}$$

The general solution is

$$\begin{aligned}
 x \cdot e^y (\cos y + \sin y) &= \int 1 \cdot e^y (\cos y + \sin y) dy + C \\
 \Rightarrow x \cdot e^y (\cos y + \sin y) &= e^y \sin y + C \left[\because \int e^x \{f(x) + f'(x)\} dx = e^x f(x) \right] \\
 \Rightarrow x (\sin y + \cos y) &= \sin y + C e^{-y}
 \end{aligned}$$

Question 27:

Solve: $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$. [*Hint* : Substitude $x+y = z$]

Answer:

$$\log \left| 1 + \tan \frac{(x+y)}{2} \right| = x + C$$