NCERT Class 12- Mathematics: Chapter – 9 Differential Equations Part 9

Long Answer (L. A.)

Question 25:

Solve: $y+\frac{\mathrm{d}}{\mathrm{d}x}(xy)=x(\sin x+\log x)$

Answer:

Given differential equation is

$$y+\frac{\mathrm{d}}{\mathrm{d}x}(xy)=x(\sin x+\log x)$$

$$\Rightarrow y+x\frac{\mathrm{d}}{\mathrm{d}x}+y=x(\sin x+\log x)\,y+x\frac{\mathrm{d}y}{\mathrm{d}x}+y=x(\sin x+\log x)$$

$$\Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=x(\sin x+\log x)$$

Dividing both sides by we get

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}+\frac{2}{x}y=\sin x+\log x$$

Which is a linear differential equation.

On comparing it with $\frac{\mathrm{d}y}{\mathrm{d}x}+Py=Q,$ we get

$$P=\frac{2}{x}, Q=\sin x+\log x$$

$$\mathrm{IF}=e^{\int\frac{2}{x}\mathrm{d}t}=e^{2\log x}=x^2$$

The complete solution is

$$y\times\mathrm{IF}=\int(\sin x+\log x)\,x^2\mathrm{d}x+C$$

The general solution is

$$y.x^2=\int(\sin x+\log x)\,x^2\mathrm{d}x+C$$

$$\Rightarrow y.x^2=\int\left(x^2\sin x+x^2\log x\right)\mathrm{d}x+C$$

$$\Rightarrow y.x^2=\int x^2\sin x\mathrm{d}x+\int x^2\log x\mathrm{d}x+C$$

$$\Rightarrow y.x^2 = I_1 + I_2 + C \ldots (i)$$

Now, $I_1 = \int x^2 \sin x\mathrm{d}x$

$$= x^2 (= \cos x) + \int 2x \cos x\mathrm{d}x$$

$$= -x^2 \cos x + \left[2x (\sin x) - \int 2 \sin x\mathrm{d}x\right]$$

$$I_1 = -x^2 \cos x + 2x \sin x + 2 \cos x \ldots (ii)$$

And $I_2 = \int x^2 \log x\mathrm{d}x$

$$= \log x.\frac{x^3}{3} - \int \frac{1}{x}.\frac{x^3}{3}\mathrm{d}x$$

$$= \log x.\frac{x^3}{3} - \frac{1}{3}.\frac{x^3}{3} \ldots (iii)$$

On substituting of value of I_1 and I_2 in Eq. (i), we get

$$y.x^2 = -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3}\log x - \frac{1}{9}x^3 + C$$

$$\therefore y = -\cos x + \frac{2 \sin x}{x} + \frac{2 \cos x}{x^2} + \frac{x}{3}\log x - \frac{x}{9} + Cx^{-2}$$

Question 26:

Find the general solution of $(1 + \tan y)(\mathrm{d}x - \mathrm{d}y) + 2x\mathrm{d}y = 0$.

Answer:

Given differential equation is $(1 + \tan y)(\mathrm{d}x - \mathrm{d}y) + 2x\mathrm{d}y = 0$

On dividing throughout by $\mathrm{d}y$, we get

$$(1 + \tan y)\left(\frac{\mathrm{d}x}{\mathrm{d}y} - 1\right) + 2x = 0$$

$$\Rightarrow (1 + \tan y)\frac{\mathrm{d}x}{\mathrm{d}y} - (1 + \tan y) + 2x = 0$$

$$\Rightarrow (1 + \tan y)\frac{\mathrm{d}x}{\mathrm{d}y} + 2x = (1 + \tan y)$$

Dividing both sides by $(1 + \tan y)$

$$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} + \frac{2x}{1 + \tan y} = 1$$

Which is a linear differential equation

On comparing it with $\frac{\mathrm{d}x}{\mathrm{d}y} + Px = Q$ we get

$$P = \frac{2}{1 + \tan y}, Q = 1$$

$$\mathrm{IF} = e^{\int \mathrm{d}y \frac{2}{1+\tan y}} e^{\int \frac{2 \cos y}{\cos y - \sin y}\mathrm{d}y}$$

$$e^{\int \frac{\cos y+\sin y+\cos y-\sin y}{\cos y-\sin y} \mathrm{d}y}$$

$$= e^{\int \left(1+\frac{\cos y-\sin y}{\cos y-\sin y}\right) \mathrm{d}y} = e^{y+\log(\cos y+\sin y)}$$

$$= e^{y}.(\cos y + \sin y) \left[\because e^{\log x} = x\right]$$

The general solution is

$$x.e^{y} (\cos y + \sin y) = \int 1.e^{y} (\cos y + \sin y) \,\mathrm{d}y + C$$

$$\Rightarrow x.e^{y} (\cos y + \sin y) = e^{y} \sin y + C \left[\because \int e^{x} \left\{f(x) + f'(x)\right\} \mathrm{d}x = e^{x} f(x)\right]$$

$$\Rightarrow x(\sin y + \cos y) = \sin y + Ce^{-y}$$

Question 27:

Solve: $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(x + y) + \sin(x + y)\,.\,[Hitn : \text{Substitude} x + y = z]$

Answer:

$$\log\left|1 + \tan\frac{(x + y)}{2}\right| = x + C$$