NCERT Class 12- Mathematics: Chapter – 9 Differential Equations Part 8

Question 17:

Find the general solution of the differential equation $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

Answer:

$$2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$$

Question 18:

Find the general solution of $y^2dx+\left(x^2-xy+y^2\right)dy=0$.

Answer:

Given, differential equation is

$$y^2dx+\left(x^2-xy+y^2\right)dy=0$$

$$\Rightarrow y^2dx=-\left(x^2-xy+y^2\right)dy$$

$$\Rightarrow y^2\frac{dx}{dy}=-\left(x^2-xy+y^2\right)$$

Dividing both sides by y^2, we get

$$\Rightarrow \frac{dx}{dy}=-\left(\frac{x^2}{y^2}-\frac{x}{y}+1\right)\ldots(i)$$

Which is a homogeneous differential equation

$$Put\frac{x}{y}=v\text{or}x=vy$$

$$\Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$$

On substituting these value in Eq. (i) , we get

$$v+y\frac{dv}{dy}=-\left[v^2-v+1\right]$$

$$\Rightarrow y\frac{dv}{dy}=-v^2-1\Rightarrow \frac{dv}{v^2+1}=-\frac{dy}{y}$$

On integrating both sides, we get

$$\int\frac{dv}{v^2+1}\int-\frac{dy}{y}$$

$$\tan^{-1}(v) = -\log y + C$$

$$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right) + \log y = C \left[\because v = \frac{x}{y}\right]$$

Question 19:

Solve: $(x+y)(dx-dy) = dx+dy$. [**Hint**: Substitute $x+y=z$ after separating dx and dy]

Answer:

$$\Rightarrow x+y = Ke^{z-y}\left[\because K = \frac{1}{C}\right]$$

Question 20:

Solve: $2(y+3) - xy\frac{dy}{dx} = 0$, given that $y(1) = -2$

Answer:

$$x^2(y+3)^3 = e^{y+2}$$

Question 21:

Solve the differential equation $dy = \cos x(2-y\operatorname{cosec} x)\ dx$ given that $y=2$ when $x = \frac{\pi}{2}$.

Answer:

$$y\sin x = \frac{-\cos 2x}{2} + \frac{3}{2}$$

Question 22:

Form the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$.

Answer:

$$xy\,y'' + x(y')^2 - yy' = 0$$

Question 23:

Solve the differential equation $(1+y^2)\tan^{-1} x dx + 2y(1+x^2)\,dy = 0$.

Answer:

$$\frac{1}{2}(\tan^{-1} x)^2 + \log(1+y^2) = c$$

Question 24:

Find the differential equation of system of concentric circles with centre $(1,2)$.

Answer:

$$(x-1)^2 + (y-2)^2 = a^2$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 4 - 4y = a^2$$

$$\Rightarrow x^2 + y^2 - 2x - 4y + 5 = a^2 \ldots (i)$$

On differentiating Eq. (i) $w.r.t.x$, we get

$$2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\Rightarrow (2x - 4)\frac{dy}{dx} + 2x - 2 = 0$$

$$\Rightarrow (y - 2)\frac{dy}{dx} + (x - 1) = 0$$