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NCERT Class 12- Mathematics: Chapter – 9 Differential Equations Part 10

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Question 28:

Find the general solution of $\frac{dy}{dx} - 3y = \sin 2x$.

Answer:

$$\Rightarrow y = -\frac{1}{13}(2\cos 2x + 3\sin 2x) + Ce^{3x}$$

Question 29:

Find the equation of a curve passing through $(2, 1)$ if the slope of the tangent to the curve at any point (x, y) is $\frac{x^2 + y^2}{2xy}$.

Answer:

It is given that, the slope of tangent to the curve at point (x, y) is $\frac{x^2 + y^2}{2xy}$

$$\therefore \left(\frac{dy}{dx}\right)_{(x,y)} = \frac{x^2 + y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right) \ldots (i)$$

Which is homogeneous differential equation.

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

On substituting these values in Eq. (i) , we get

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v} + v\right)$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1 + v^2}{v}\right)$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$\Rightarrow \frac{2v}{1+v^2}\mathrm{d}v = \frac{\mathrm{d}x}{x}$$

On integrating both sides, we get

$$\int \frac{2v}{1+v^2}\mathrm{d}v = \int \frac{\mathrm{d}x}{x}$$

Put $1 - v^2 = t$ in LHS, we get

$$-2v\mathrm{d}v = \mathrm{d}t$$

$$\Rightarrow -\int \frac{\mathrm{d}t}{t} = \int \frac{\mathrm{d}x}{x}$$

$$\Rightarrow -\log t = \log x + \log C$$

$$\Rightarrow -\log\left(1 - v^2\right) = \log x + \log C$$

$$\Rightarrow -\log\left(1 - \frac{y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow -\log\left(\frac{x^2 - y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow \frac{x^2}{x^2 - y^2} = Cx$$

Since, the curve passes through the point $(2, 1)$

$$\therefore \frac{(2)^2}{(2)^2 - (1)^2} = C(2) \Rightarrow C = \frac{2}{3}$$

So, the required solution is $2\left(x^2 - y^2\right) = 3x$

Question 30:

Find the equation of the curve through the point (1,0) if the slope of tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$

Answer:

It is given that, slope of tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$

$$\therefore \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(x,y)} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \frac{\mathrm{d}y}{y-1} = \frac{\mathrm{d}x}{x^2+x}$$

On integrating both sides, we get

$$\int \frac{\mathrm{d}y}{y-1} = \int \frac{\mathrm{d}x}{x^2+x}$$

$$\Rightarrow \int \frac{\mathrm{d}y}{y-1} = \int \frac{\mathrm{d}x}{x(x+1)}$$

$$\Rightarrow \int \frac{dy}{y-1} = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$$

$$\Rightarrow \log(y-1) = \log x - \log(x+1) + \log C$$

$$\Rightarrow \log(y-1) = \log\left(\frac{xC}{x+1}\right)$$

Since, the given curve passes through point $(1, 0)$

$$\therefore 0 - 1 = \frac{1.C}{1+1} \Rightarrow C = -2$$

The particular solution is $y - 1 = \frac{-2x}{x+1}$

$$\Rightarrow (y-1)(x+1) = -2x$$

$$\Rightarrow (y-1)(x+1) + 2x = 0$$