NCERT Class 12- Mathematics: Chapter – 8 Application of Integrals Part 1

8.1 Overview

This chapter deals with a specific application of integrals to find the area under simple curves, area between lines and arcs of circles, parabolas and ellipses, and finding the area bounded by the above said curves.

8.1. 1 The area of the region bounded by the curve $y = f(x)$, $x-$ axis and the lines $x = a$ and $x = b\,(b > a)$ is given by the formula:

$$Area = \int_a^b y\mathrm{d}x = \int_a^b (x)\,\mathrm{d}x$$

8.1. 2 The area of the region bounded by the curve $x = \phi(y), y-$ axis and the lines $y = c, y = d$ is given by the formula:

$$Area = \int_c^d x\mathrm{d}y = \int_c^d (y)\,\mathrm{d}y$$

8.1. 3 The area of the region enclosed between two curves $y = f(x), y = g(x)$ and the lines $x = a, x = b$ is given by the formula.

$$Area = \int_c^d f(x) - g(x)\,\mathrm{d}x, \text{where} f(x).g(x)\,\text{in}\,[a, b]$$

8.1. 4 $\text{If} f(x)\,g(x)\,\text{in}\,[a, c]\,\text{and} f(x)\,g(x)\,\text{in}\,[c, b], a < c < b$, then

$$Area = \int_a^c f(x) - g(x)\,\mathrm{d}x \int_c^b g(x) - f(x)\,\mathrm{d}x$$

8.2 Solved Examples

Short Answer (S. A)

Question 1:

Find the area of the curve $y = \sin x$ between and .

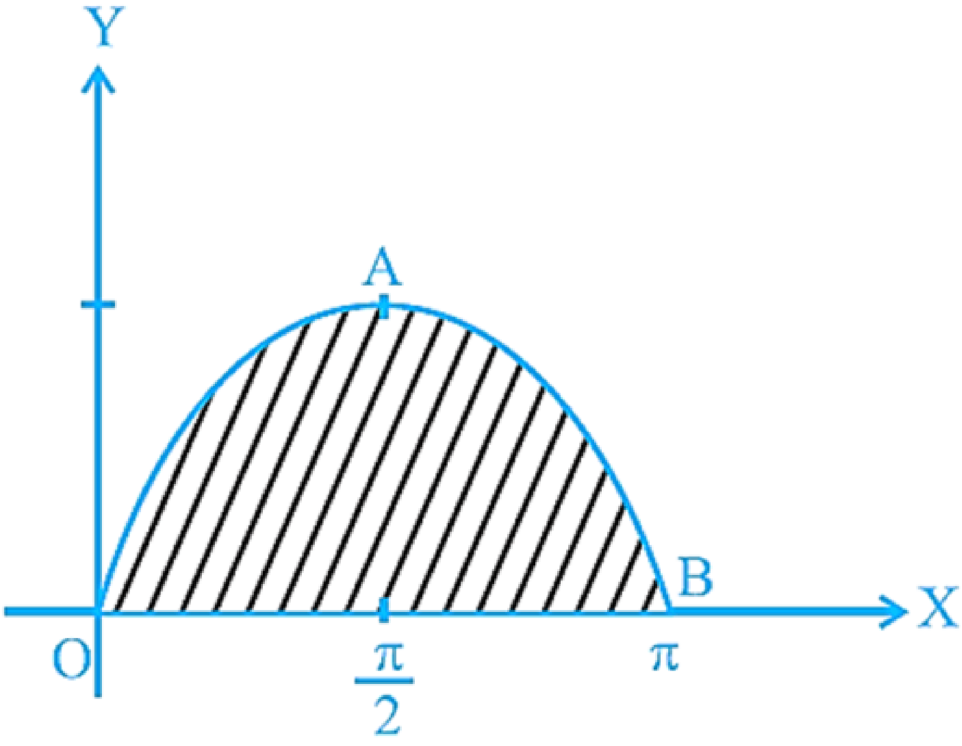

Fig. 8.1

Answer:

$$\text{Area OAB} = \int_0^{\pi} y\text{d}x = \int_0^{\pi} \sin x\text{d}x = |-\cos x|_0^{\pi}$$

$$= \cos 0 - \cos \pi = 2\text{sq units.}$$

Question 2:

Find the area of the region bounded by the curve $ay^2 = x^3$, the $y-$ axis and the lines $y = a$ and $y = 2a$.

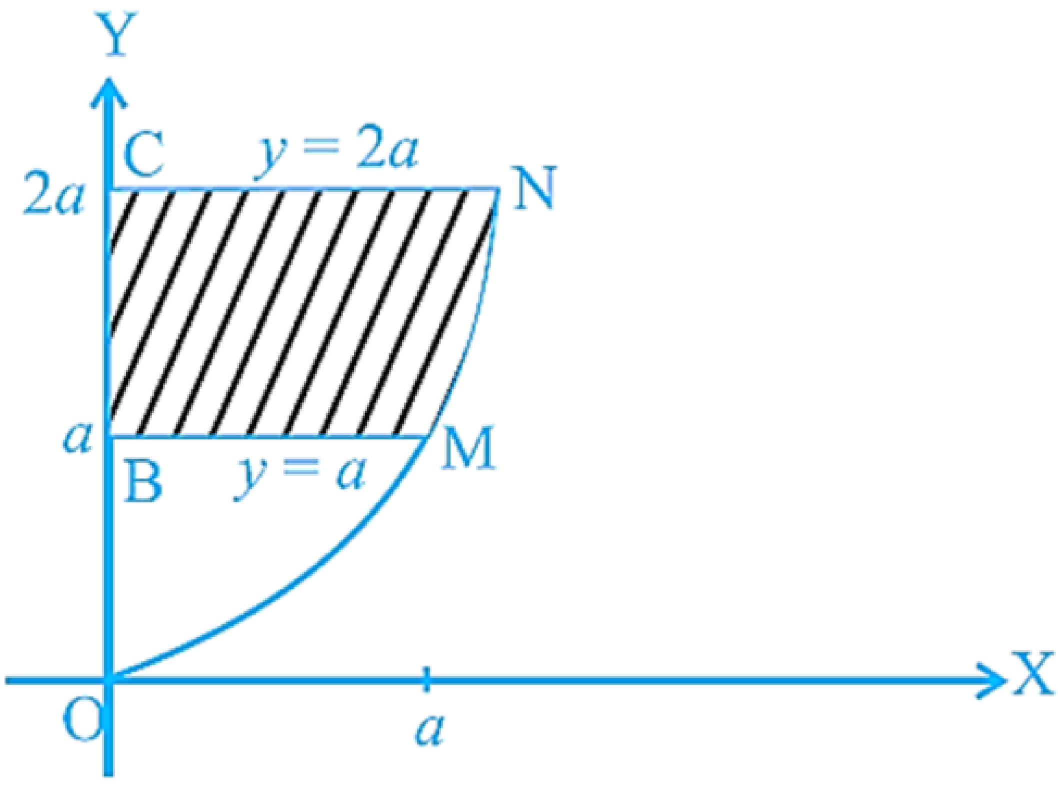

Fig. 8.2

Answer:

$$\text{Area BMNC} = \int_a^{2a} y\mathrm{d}x = \int_a^{2a} a^{\frac{1}{3}} y^{\frac{2}{3}} \mathrm{d}y$$

$$= \frac{3a^{\frac{1}{3}}}{5} \left| y^{\frac{5}{3}} \right|_a^{2a}$$

$$= \frac{3a^{\frac{1}{3}}}{5} \left| 2^{a^{\frac{5}{3}}} - a^{\frac{5}{3}} \right|$$

$$= \frac{3}{5} a^{\frac{1}{3}} - a^{\frac{5}{3}} \left| (2)^{\frac{5}{3}-1} \right|$$

$$= \frac{3}{5} a^2 \left| 2.2^{\frac{2}{3}} - 1 \right| \text{sq units.}$$

Question 3:

Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$.

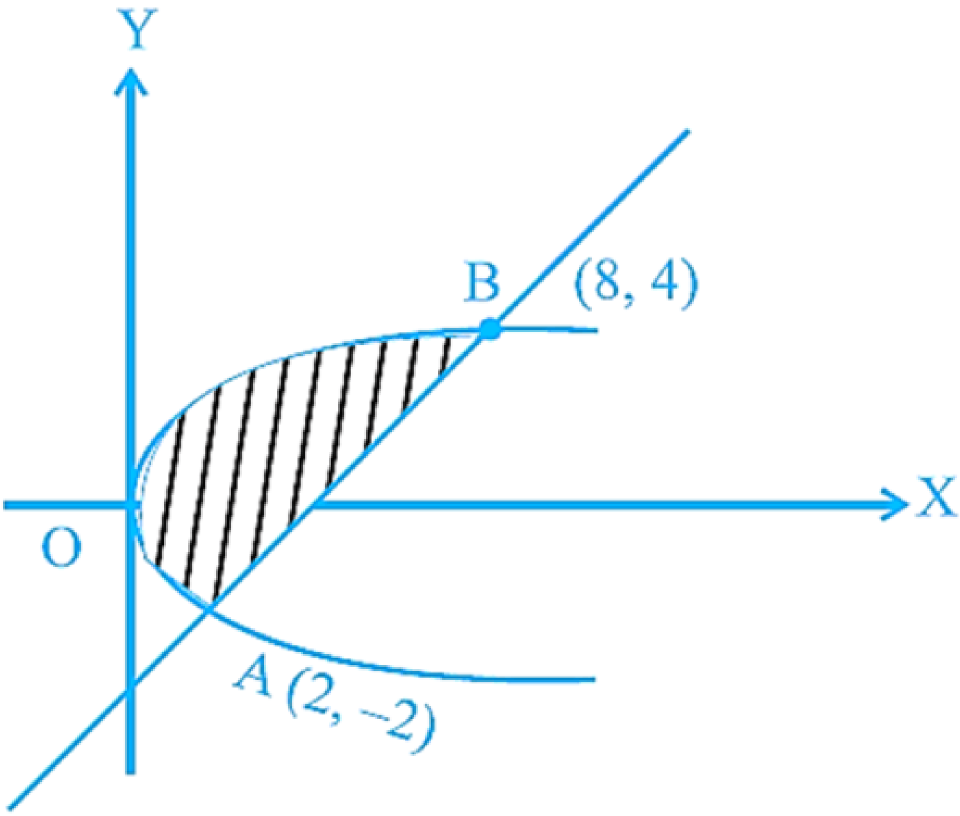

Fig. 8.3

Answer:

The intersecting points of the given curves are obtained by solving the equations $x - y = 4$ and $y^2 = 2x$ for and .

We have $y^2 = 8 + 2y \, i.e., (y - 4)(y + 2) = 0$ which gives $y = 4, -2$ and $x = 8, 2$.

Thus, the points of intersection are $(8, 4), (2, -2)$. Hence

$$Area = \int_{-2}^{4} \left(4 - y - \frac{1}{2}y^2\right) \mathrm{d}y$$

$$= \left| 4y + \frac{y^2}{2} - \frac{1}{6}y^3 \right|_{-2}^{4} = 18 \text{sq units.}$$