NCERT Class 12- Mathematics: Chapter – 5 Continuity and Differentiability Part 1

5.1 Overview

5.1. 1 Continuity of a function at a point

Let f be a real function on a subset of the real numbers and let be a point in the domain of f. Then f is continuous at if

$$\lim_{x \to c} f(x) = f(c)$$

More elaborately, if the left hand limit, right hand limit and the value of the function at $x = c$ exist and are equal to each other, i.e.. ,

$$\lim_{x \to c} f(x) = f(c) \lim_{x \to c} f(x)$$

Then f is said to be continuous at $x = c$.

5.1. 2 Continuity in an interval

(i) f is said to be continuous in an open interval (a, b) if it is continuous at every point in this interval.

(ii) f is said to be continuous in the closed interval $[a, b]$ if

- f is continuous in (a, b)
- $\lim_{x \to a^+} f(x) = f(a)$
- $\lim_{x \to a^-} f(x) = f(b)$

5.1. 3 Geometrical meaning of continuity

(i) Function f will be continuous at $x = c$ if there is no break in the graph of the function at the point $(c, f(c))$.

(ii) In an interval, function is said to be continuous if there is no break in the graph of the function in the entire interval.

5.1. 4 Discontinuity

The function f will be discontinuous at $x = a$ in any of the following cases:

(i) $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist but are not equal.

(ii) $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist and are equal but not equal to $f(a)$.

(iii) $f(a)$ is not defined.

5.1. 5 Continuity of some of the common functions

	Function $f(x)$	**Interval in which is continuous**
1.	The constant function, i.e.. $f(x) = c$	R
2.	The identity function, i.e.. $f(x) = x$	R
3.	The polynomial function, i.e.. $f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n$	R
4.	$\lvert x - a \rvert$	$(-\infty, \infty)$
5.	x^n, n is a positive integer	$(-\infty, \infty) - \{0\}$
6.	$\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials in	$R - \{x : q(x) = 0\}$
7.	$\sin x, \cos x$	R
8.	$\tan x, \sec x$	$R - \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$
9.	$\cot x, co\sec x$	$R - \{n\pi : n \in Z\}$
10.	e^x	R
11	$\log x$	$(0, \infty)$
12	The inverse trigonometric functions, $i.e., \sin^{-1} x, \cos^{-1} x\ etc$	In their respective domains

Continuity of Some of the Common Functions

5.1. 6 Continuity of composite functions

Let and be real valued functions such that (fog) is defined at a. If is continuous at a and f is continuous at $g(a)$, then (fog) is continuous at a.

5.1. 7 Differentiability

The function defined by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, wherever the limit exists, is defined to be the derivative of at . In other words, we say that a function is differentiable at a point in its domain if both $\lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$, called left hand derivative, denoted by $Lf'(c)$, and $\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$, called right hand derivative, denoted by $Rf'(c)$, are finite and equal.

(i) The function $y = f(x)$ is said to be differentiable in an open interval (a, b) if it is differentiable at every point of (a, b)

(ii) The function $y = f(x)$ is said to be differentiable in the closed interval $[a, b]$ if $Rf'(a)$ and $Lf'(b)$ exist and $f'(x)$ exists for every point of (a, b) .

(iii) Every differentiable function is continuous, but the converse is not true

5.1. 8 Algebra of derivatives

If u, v are functions of , then

(i) $\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

(ii) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

(iii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

5.1. 9 Chain rule is a rule to differentiate composition of functions. Let $f = vou$. If

$$i = u(x) \text{ and both} \frac{dt}{dx} \text{and} \frac{dv}{dt} \text{exist then} \frac{df}{dx} = \frac{dv}{dt}.\frac{dt}{dx}$$

5.1. 10 Following are some of the standard derivatives (in appropriate domains)

1. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1 - x^2}}$

2. $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1 - x^2}}$

3. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1 - x^2}$

4. $\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1 - x^2}$

5. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{|x|\sqrt{x^2 - 1}}, |x| > 1$

6. $\frac{d}{dx}\left(\cos ec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2 - 1}}, |x| > 1$