FlexiPrep

NCERT Class 12-Mathematics: Chapter – 13 Probability Part 1 (For CBSE, ICSE, IAS, NET, NRA 2022)

13.1 Overview

13.1. 1 Conditional Probability

If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the event E under the condition that the event F has occurred, written as $P(E|F)$, is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

13.1. 2 Properties of Conditional Probability

Let E and F be events associated with the sample space S of an experiment. Then:

(i) $P(S|F) = P(F|F) = 1$

(ii) $P[(A \cup B)|F] = P(A|F) + P(B|F) - P[(A \cap B|F)]$, where A and B are any two events associated with S.

(iii) $P\left(E'\middle|F\right) = 1 - P(E|F)$

13.1. 3 Multiplication Theorem on Probability

Let E and F be two events associated with a sample space of an experiment. Then

$$P(E \cap F) = P(E)\,P(F|E), P(E) \neq 0$$

$$= P(F)\,P(E|F), P(F) \neq 0$$

If E, F and G are three events associated with a sample space, then

$$P(E \cap F \cap G) = P(E)\,P(F|E) P(G\,|E \cap F)$$

13.1. 4 Independent Events

Let E and F be two events associated with a sample space S. If the probability of occurrence of one of them is not affected by the occurrence of the other, then we say that the two events are independent. Thus, two events E and F will be independent, if

(a) $P(F|E) = P(F)$ provided $P(E) \neq 0$

(b) $P(E|F) = P(E)$, provided $P(F) \neq 0$

Using the multiplication theorem on probability, we have

(c) $P(E \cap F) = P(E)\,P(F)$

Three events A, B and C are said to be mutually independent if all the following conditions hold:

$$P(A \cap B) = P(A)\,P(B)$$

$$P(A \cap C) = P(A)\,P(C)$$

$$P(B \cap C) = P(B)\,P(C)$$

and $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

13.1. 5 Partition of a Sample Space

A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of a sample space S if

(a) $E_i \cap E_j = \phi, i \neq j; i, j = 1,2,3, \ldots, n$

(b) $E_i \cup E_2 \cup \ldots \cup E_n = S$, and

(c) Each $E_i \neq \phi, i.e, P(E_i) > 0$ for all $i = 1,2,\ldots,n$

13.1. 6 Theorem of Total Probability

Let $\{E_1, E, \ldots, E_n\}$ be a partition of the sample space S. Let A be any event associated with S, then

$$P(A) = \sum_{j-1}^{n} P\left(E_j\right) P\left(A|E_j\right)$$

13.1. 7 Bayes' Theorem

If $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events associated with a sample space, and A is any event of non-zero probability, then

$$P(E|A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{i-1}^{n} P(E_i)\,P(A|E_i)}$$

13.1. 8 Random Variable and its Probability Distribution

A random variable is a real valued function whose domain is the sample space of a random experiment.

The probability distribution of a random variable X is the system of numbers

$X:$	x_1		x_2	$\ldots x_n$
$P(X):$	p_1		p_2	$\ldots p_n$

Random Variable and Its Probability Distribution

Where $p_i > 0, i = 1, \ldots, n, \sum_{i-1}^{n} P_i = 1$

13.1. 9 Mean and Variance of a Random Variable

Let X be a random variable assuming values $x_1, x_2 \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$, respectively such that $p_i \geqslant 0, \sum_{i-1}^{n} P_i = 1$. Mean of X, denoted by μ [expected value of X denoted by $E(X)$] is defined as

$$\mu = E(X) = \sum_{i-1}^{n} x_i P_i$$

and variance, denoted by σ^2, is defined as

$$\sigma^2 = \sum_{i-1}^{n} (x_i - \mu)^2 p_i = \sum_{i-1}^{n} x_i^2 p_i = \mu^2 \text{ or equivalently}$$

$$\sigma^2 = E(X - \mu)^2$$

Standard deviation of the random variable X is defined as

$$= \sqrt{\text{variance}(X)} = \sqrt{\sum_{i-1}^{n} (x_i - \mu)^2 p_i}$$

13.1. 10 Bernoulli Trials

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials

(ii) The trials should be independent

(iii) Each trial has exactly two outcomes: success or failure

(iv) The probability of success (or failure) remains the same in each trial.

13.1. 11 Binomial Distribution

A random variable X taking values $0,1,2,\ldots,n$ is said to have a binomial distribution with parameters n and p, if its probability distribution is given by

$$P(X = r) = {}^{n}c_r\, p^r q^{n-r},$$

$$\text{where } q = 1 - p \text{ and } r = 0,1,2,\ldots,n.$$