NCERT Class 12- Mathematics: Chapter – 10 Vector Algebra Part 1

10.1 Overview

10.1. 1 A quantity that has magnitude as well as direction is called a vector.

10.1. 2 The unit vector in the direction of $\vec{a}$ is given by $\frac{\vec{a}}{|\vec{a}|}$ and is represented $\hat{a}$.

10.1. 3 Position vector of a point $P(x, y, z)$ is given as $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude as $\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2}$, where O is the origin.

10.1. 4 The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.

10.1. 5 The magnitude r, direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as:

$$l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}.$$

10.1. 6 The sum of the vectors representing the three sides of a triangle taken in order is $\vec{0}$

10.1. 7 The triangle law of vector addition states that "If two vectors are represented by two sides of a triangle taken in order, then their sum or resultant is given by the third side taken in opposite order" .

10.1. 8 Scalar multiplication

If $\vec{a}$ is a given vector and λ a scalar, then $\lambda\vec{a}$ is a vector whose magnitude is $\left|\lambda\vec{a}\right| = |\lambda|\left|\vec{a}\right|$. The direction of $\lambda\vec{a}$ is same as that of $\vec{a}$ if λ is positive and, opposite to that of $\vec{a}$ if λ is negative.

10.1. 9 Vector joining two points

If $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are any two points, then

$$\overrightarrow{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\overrightarrow{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

10.1. 10 Section formula

The position vector of a point R dividing the line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$

(i) In the ratio $m : n$ internally, is given by $\left|\frac{n\vec{a} + m\vec{b}}{m + n}\right|$

(ii) in the ratio $m : n$ externally, is given by $\left|\frac{m\vec{b} - n\vec{a}}{m-n}\right|$

10.1. 11 Projection of $\vec{a}$ along $\vec{b}$ is $\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}$ and the Projection vector of $\vec{a}$ along $\vec{b}$ is $\left(\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}\right)\vec{b}$.

10.1. 12 Scalar or dot product

The scalar or dot product of two given vectors $\vec{a}$ and $\vec{a}$ having an angle θ between them is defined as

$$\vec{a}.\vec{b} = \left|\vec{a}\right|.\left|\vec{b}\right|\cos\theta$$

10.1. 13 Vector or cross product

The cross product of two vectors $\vec{a}$ and $\vec{b}$ having angle θ between them is given as

$$\vec{a} \times \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta\hat{n},$$

where is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$ and $\vec{a}, \vec{b}$, , form a right-handed system?

10.1. 14 If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two vectors and is any scalar, then

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$$

$$\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = (b_1c_2 - b_2c_1)\hat{i} + (a_2c_1 - c_1c_2)\hat{j} + (a_1b_b - a_2b_1)\hat{k}$$

Angle between two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|.\left|\vec{b}\right|} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

10.2 Solved Examples

Short Answer (S. A)

Question 1:

Find the unit vector in the direction of the sum of the vectors

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \text{and} \vec{b} = -\hat{i} + \hat{j} + 3\hat{k}.$$

Answer:

Let $\vec{c}$ denote the sum of $\vec{a}$ and $\vec{b}$ We have

$$\vec{c} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \left(-\hat{i} + \hat{j} + 3\hat{k}\right) = -\hat{i} + 5\hat{k}$$

Now $\left|\vec{c}\right| = \sqrt{1^2 + 5^2} = \sqrt{26}$.

Thus, the required unit vector is $\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{26}}\left(\hat{i} + 5\hat{k}\right) = \frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{k}$

Question 2:

Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$ where P and Q are the points $(1, 3, 2)$ and $(-1, 0, 8)$, respectively.

$$\overrightarrow{QP} = (-1-1)\,\hat{i}\,(0-3)\,\hat{j} + (8-2)\,\hat{k} = -2\hat{i} - 3\hat{j} + 6\hat{k}$$

Thus $\overrightarrow{QP} = -\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$$\Rightarrow \left|\overrightarrow{QP}\right| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$$

Therefore, unit vector in the direction of $\overrightarrow{QP}$ is given by

$$\widehat{QP}\,\frac{\overrightarrow{QP}}{\left|\overrightarrow{QP}\right|} = \frac{2\hat{i}3\hat{j}6\hat{k}}{7}$$

Hence, the required vector of magnitude 11 in direction of $\overrightarrow{QP}$ is

$$11\,\widehat{QP} = 11\frac{2\hat{i}3\hat{j}6\hat{k}}{7} = \frac{22}{7}\hat{i} + \frac{33}{7}\hat{j} - \frac{66}{7}\hat{k}.$$