FlexiPrep

NCERT Class 11-Math's: Chapter – 9 Sequence and Series Part 1 (For CBSE, ICSE, IAS, NET, NRA 2022)

9.1 Overview:

By a sequence, we mean an arrangement of numbers in a definite order according to some rule. We denote the terms of a sequence by $a_1, a_2, a_3, \ldots$, etc. , the subscript denotes the position of the term.

In view of the above a sequence in the set X can be regarded as a mapping or a function $f : N \rightarrow X$ defined by

$$f(n) = t_n \forall n \in N.$$

Domain of f is a set of natural numbers or some subset of it denoting the position of term. If its range denoting the value of terms is a subset of R real numbers then it is called a real sequence.

A sequence is either finite or infinite depending upon the number of terms in a sequence. We should not expect that its terms will be necessarily given by a specific formula.

However, we expect a theoretical scheme or rule for generating the terms.

Let a_1, a_2, a_3 , ... , be the sequence, then, the expression $a_1 + a_2 + a_3 + \ldots$ is called the series associated with given sequence. The series is finite or infinite according as the given sequence is finite or infinite.

Remark When the series is used, it refers to the indicated sum not to the sum itself. Sequence following certain patterns are more often called progressions. In progressions, we note that each term except the first progresses in a definite manner.

9.1. 1 Arithmetic progression (A. P.)

Is a sequence in which each term except the first is obtained by adding a fixed number (positive or negative) to the preceding term.

Thus any sequence $a_1, a_2, a_3 \ldots$ an, $\ldots$ is called an arithmetic progression if $a_{n+1} + a_n + d, n \in N$, where d is called the common difference of the A. P. , usually we denote the first term of an A. P by a and the last term by l

The general term or the n^{th} term of the A. P. is given by

$$a_n = a + (n - 1) d$$

The nth term from the last is given by $a_n = l-(n-1)\,d$

The sum S_n of the first n terms of an A. P. is given by

$S_n = \frac{n}{2}[2a + (n-1)\,d] = \frac{n}{2}(a + l)$, where $l = a + (n-1)\,d$ is the last terms of the A. P. , and the general term is given by $a_n = S_n - S_{n-1}$

The arithmetic mean for any n positive numbers $a_1, a_2, a_3, \ldots a_n$ is given by

$$A.M. = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

If a , A and b are in A. P. , then A is called the arithmetic mean of numbers a and b and i.e.. ,

$$A = \frac{a+b}{2}$$

If the terms of an A. P. are increased, decreased, multiplied or divided by the same constant, they still remain in A. P.

If $a_1, a_2, a_3 \ldots$ are in A. P. with common difference d , then

(i) $a_1 \pm k, a_2 \pm k, a_3 \pm k, \ldots$ are also in A. P with common difference d .

(ii) $a_1\ k, a_2\ k, a_3\ k, \ldots$ are also in A. P with common difference dk $(k \neq 0)$.

And $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k} \ldots$ are also in A. P. with common difference $\frac{d}{k}(k \neq 0)$

If $a_1, a_2, a_3 \ldots$ and $b_1, b_2, b_3 \ldots$ are two A. P. , then

(i) $a_1 \pm a_1, a_2 \pm a_2, a_3 \pm b_3 =$ __________ are also in A. P

(ii) $a_1 b_1, a_2\ b_2, a_3\ b_3$, __________and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$, ... are not in A. P.

If a_1, a_2, a_3 ... and a_n are in A. Ps, then

(i) $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} =$ __________

(ii) $a_r = \frac{a_{r-k} + a_{r+k}}{2} \forall k, 0 \leqslant k \leqslant n-r$

(iii) If n^{th} term of any sequence is linear expression in n , then the sequence is an A. P.

(iv) If sum of n terms of any sequence is a quadratic expression in n , then sequence is an A. P.

9.1. 2 A Geometric progression (G. P.)

Is a sequence in which each term except the first is obtained by multiplying the previous term by a non-zero constant called the common ratio. Let us consider a G. P. with first non-zero term a and common ratio r , i.e.. ,

$$a, \mathrm{ar}, \mathrm{ar}^2, \ \ldots \ \mathrm{ar}^{n-1}, \ \ldots$$

Here, common ratio $r = \dfrac{\mathrm{ar}^{n-1}}{\mathrm{ar}^{n-2}}$

The general term or n^{th} term of G. P. is given by $a_n = \mathrm{ar}^{n-1}$.

Last term l of a G. P. is same as the nth term and is given by $l = \mathrm{ar}^{n-1}$

and the n^{th} term from the last is given by $a_n = \dfrac{l}{r^{n-1}}$

The sum S_n of the first n terms is given by

$$S_n = \frac{a\,(r^n - 1)}{r - 1}, \text{if } r \neq 1$$

$$S_n = \text{na if } r = 1$$

If a , G and b are in G. P. , then G is called the geometric mean of the numbers a and b and is given by

$$G = \sqrt{\mathrm{ab}}$$

(i) If the terms of a G. P. are multiplied or divided by the same non-zero constant $(k \neq 0)$, they still remain in G. P.

If $a_1, a_2, a_3, \ \ldots$, are in GP, then $a_1\ k, a_2\ k, a_3\ k, \ldots$ and $\dfrac{a_1}{k}, \dfrac{a_2}{k}, \dfrac{a_3}{k}$, ... are also in G. P. with same common ratio, in particularly

If $a_1, a_2, a_3, \ \ldots$, are in GP, then

$\dfrac{1}{a_1}, \dfrac{1}{a_2}, \dfrac{1}{a_3}, \ \ldots$ Are also in G. P.

(ii) If $a_1, a_2, a_3, \ \ldots$ and b_1, b_2, b_3, ... are two G. P. s, then $a_1\ b_1, a_2\ b_2, a_3\ b_3\ \ldots$ And $\dfrac{a_1}{b_1}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_2}$, ... are also in G. P.

(iii) If $a_1, a_2, a_3, \ \ldots$ Are in A. P. $(a_i > 0 \forall i)$, then $x^{a_1}, x^{a_2}, x^{a_3}, \ldots$, are in G. P. $(\forall x > 0)$

(iv) If $a_1, a_2, a_3, \ldots, a_n$ are in G. P. , then $a_1\ a_n = a_2\ a_{n-1} = a_3\ a_{n-2} = \ldots$