NCERT Class 11- Math's: Chapter – 12 Introduction to Three Dimensional Geometry Part 1

12.1 Overview

12.1. 1 Coordinate axes and coordinate planes

Let X′OX, Y′OY, Z′OZ be three mutually perpendicular lines that pass through a point O such that X′OX and Y′OY lies in the plane of the paper and line Z′OZ is perpendicular to the plane of paper. These three lines are called rectangular axes (lines X′OX, Y′OY and Z′OZ are called *x*-axis, *y*-axis and *z*-axis) . We call this coordinate system a three dimensional space, or simply space.

The three axes taken together in pairs determine xy, yz, zx -plane , i.e.. , three coordinate planes. Each plane divide the space in two parts and the three coordinate planes together divide the space into eight regions (parts) called octant, namely (i) OXYZ (ii) OX′YZ (iii) OXY′Z (iv) OXYZ′ (v) OXY′Z′ (vi) OX′YZ′ (vii) OX′Y′Z (viii) OX′Y′Z′. (Fig. 12.1)

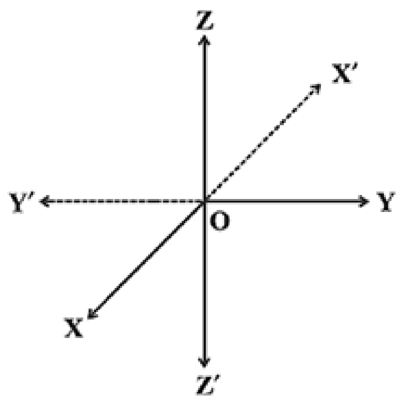

Fig. 12.1

Let P be any point in the space, not in a coordinate plane, and through P pass planes parallel to the coordinate planes yz, zx and xy meeting the coordinate axes in the points A, B, C respectively.

Three planes are

(i) ADPF $|| yz-$ plane

(ii) BDPE $|| xz-$ plane

(iii) CFPE $|| xy-$ plane

These planes determine a rectangular parallelepiped which has three pairs of rectangular faces

(A D P F, O B E C) , (B D P E, C F A O) and (A O B D, FPEC) (Fig 12.2)

12.1. 2 Coordinate of a point in space

An arbitrary point P in three-dimensional space is assigned coordinates (x_0, y_0, z_0) provided that

(1) The plane through P parallel to the $yz-$ plane intersects the *x*-axis at $(x_0, 0, 0)$;

(2) The plane through P parallel to the $xz-$ plane intersects the *y*-axis at $(0, y_0, 0)$;

(3) The plane through P parallel to the $xy-$ plane intersects the *z*-axis at $(0, 0, z_0)$.

The space coordinates (x_0, y_0, z_0) are called the Cartesian coordinates of P or simply the rectangular coordinates of P.

Moreover we can say, the plane ADPF (Fig. 12.2) is perpendicular to the *x*-axis or xaxis is perpendicular to the plane ADPF and hence perpendicular to every line in the plane.

Therefore, PA is perpendicular to OX and OX is perpendicular to PA. Thus A is the foot of perpendicular drawn from P on *x*-axis and distance of this foot A from O is *x*-coordinate of point P. Similarly, we call B and C are the feet of perpendiculars drawn from point P on the *y* and *z*-axis and distances of these feet B and C from O are the *y* and *z* coordinates of the point P.

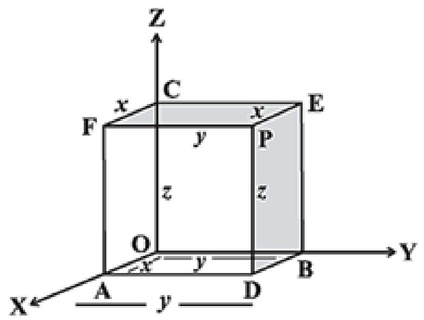

Fig. 12.2

Hence the coordinates $x, y\ z$ of a point P are the perpendicular distance of P from the three coordinate planes yz, zx and xy , respectively.

12.1. 3 Sign of coordinates of a point

The distance measured along or parallel to OX, OY, OZ will be positive and distance moved along or parallel to OX′, OY′, OZ′ will be negative. The three mutually perpendicular coordinate plane which in turn divide the space into eight parts and each part is know as octant. The sign of the coordinates of a point depend upon the octant in which it lies. In first octant all the coordinates are positive and in seventh octant all coordinates are negative. In third octant x, y coordinates are negative and z is positive. In fifth octant x, y are positive and z is negative. In fourth octant x, z are positive and y is negative. In sixth octant x, z are negative y is positive. In the second octant x is negative and y and z are positive.

Octants → Coordinates	OXYZ	OX ′ YZ	III OX 'Y' Z	IV OXY ′ Z	V OXYZ ′	VI OX 'YZ'	VII OX ′ Y ′ Z ′	VIII OXY 'Z'
	+ + +	− + +	− − +	+ − +	+ + −	− + −	− − −	+ − −

Octants Coordinates

12.1. 4 Distance formula:

The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

A parallelepiped is formed by planes drawn through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) parallel to the coordinate planes. The length of edges are $x_2 - x_1, y_2 - y_1, z_2 - z_1$ and length of diagonal is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_1 - z_1)^2}$.

12.1. 5 Section formula:

The coordinates of the point R which divides the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally or externally in the ratio $m : n$ are given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right), \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right), \text{respectively}$$

The coordinates of the mid-point of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1) , $(x_2, y_2, z_2)\, x_3, y_3, z_3$ are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$.