NCERT Class 11- Math's: Exemplar Chapter – 11 Conic Sections Part 1

11.1 Overview

11.1. 1 Sections of a cone

Let be a fixed vertical line and $_m$ be another line intersecting it at a fixed point V and inclined to it at an angle α (Fig. 11.1) .

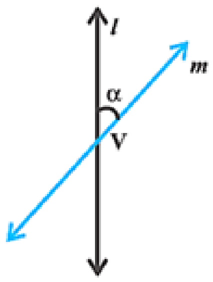
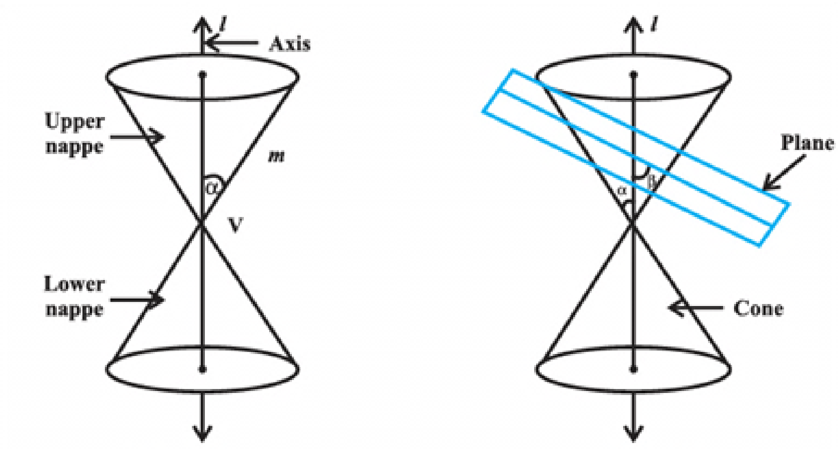


Fig. 11.1

Suppose we rotate the line *m* around the line *l* in such a way that the angle α remains constant. Then the surface generated is a double-napped right circular hollow cone herein after referred as cone and extending indefinitely in both directions (Fig. 11.2) .



The point V is called the vertex; the line l is the axis of the cone. The rotating line m is called a generator of the cone. The vertex separates the cone into two parts called nappes.

If we take the intersection of a plane with a cone, the section so obtained is called a conic section. Thus, conic sections are the curves obtained by intersecting a right circular cone by a plane.

We obtain different kinds of conic sections depending on the position of the intersecting plane with respect to the cone and the angle made by it with the vertical axis of the cone. Let β be the angle made by the intersecting plane with the vertical axis of the cone (Fig. 11.3) .

The intersection of the plane with the cone can take place either at the vertex of the cone or at any other part of the nappe either below or above the vertex.

When the plane cuts the nappe (other than the vertex) of the cone, we have the following situations:

(a) When $\beta = 90^o$, the section is a circle.

(b) When $\alpha < \beta < 90^o$, the section is an ellipse.

(c) When $\beta = \alpha$; the section is a parabola.

(In each of the above three situations, the plane cuts entirely across one nappe of the cone) .

(d) When $0 \leqslant \beta < \alpha$; the plane cuts through both the nappes and the curves of intersection is a *hyperbola*.

Indeed these curves are important tools for present day exploration of outer space and also for research into the behaviour of atomic particles.

We take conic sections as plane curves. For this purpose, it is convenient to use equivalent definition that refer only to the plane in which the curve lies, and refer to special points and lines in this plane called foci and directories. According to this approach, parabola, ellipse and hyperbola are defined in terms of a fixed point (called focus) and fixed line (called directrix) in the plane.

If S is the focus and *l* is the directrix, then the set of all points in the plane whose distance from S bears a constant ratio *e* called eccentricity to their distance from *l* is a conic section.

As special case of ellipse, we obtain circle for which $e = 0$ and hence we study it differently.

11.1. 2 Circle

A circle is the set of all points in a plane which are at a fixed distance from a fixed point in the plane. The fixed point is called the centre of the circle and the distance from centre to any point on the circle is called the radius of the circle.

The equation of a circle with radius *r* having centre (h, k) is given by $(x-h)^2 + (y-k)^2 = r^2$ The general equation of the circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$, where *g*, *f* and *c* are constants.

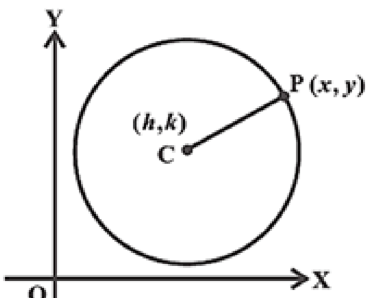


Fig. 11.4

(a) The centre of this circle is $(-g, -f)$

(b) The radius of the circle is $\sqrt{g^2 + f^2 - c}$

The general equation of the circle passing through the origin is given by $x^2 + y^2 + 2gx + 2fy = 0$.

General equation of second degree i.e.. , $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a circle if (i) the coefficient of x^2 equals the coefficient of y^2 , i.e.. , $a = b \neq 0$ and (ii) the coefficient of xy is zero, i.e.. , $h = 0$.

The parametric equations of the circle $x^2 + y^2 = r^2$ are given by $x = r\cos\theta, y = r\sin\theta$ where is the parameter and the parametric equations of the circle $(x-h)^2 + (y-k)^2 = r^2$ are given by

$$x-h = r\cos\theta, y-k = r\sin\theta$$

or $x = h + r\cos\theta, y = k + r\sin\theta.$

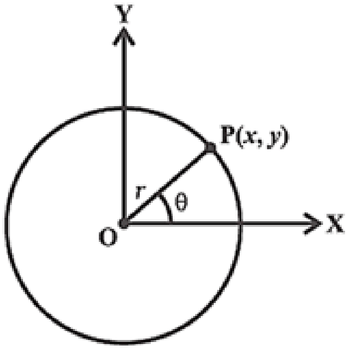


Fig. 11.5

Note: The general equation of the circle involves three constants which implies that at least three conditions are required to determine a circle uniquely.