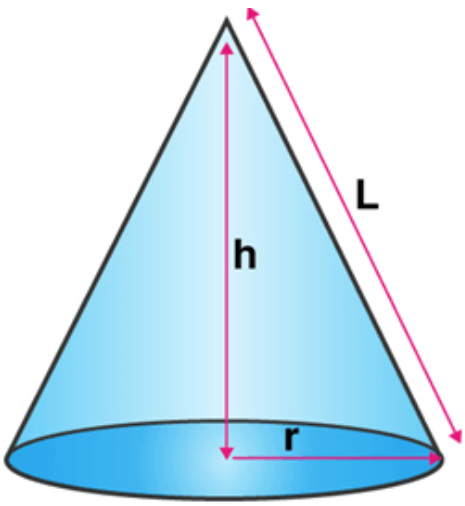
Volume of a Cone: Formula for Volume of a Cone and Derivation of Cone Volume

The **volume of a cone** defines the space or the capacity of the cone. A **cone** is a three-dimensional geometric shape having a circular base that tapers from a flat base to a point called **apex or vertex**. A cone is formed by a set of line segments, half-lines or lines connecting a common point, the apex, to all of the points on a base that is in a plane that does not contain the apex.

Formula for Volume of a Cone

Cone is a pyramid with a circular cross-section. A right cone is a cone with its vertex above the surface. When it is not mentioned as a 'cone' is referred to as a 'right-cone' or right circular cone.



Therefore, the volume of a cone formula is given as

$$\textit{The volume of a cone} = \left(\frac{1}{3}\right)\pi r^2 h \ \textit{cubic units}$$

Where,

$'r'$ is the base radius of the cone

$'l'$ is the slant height of a cone

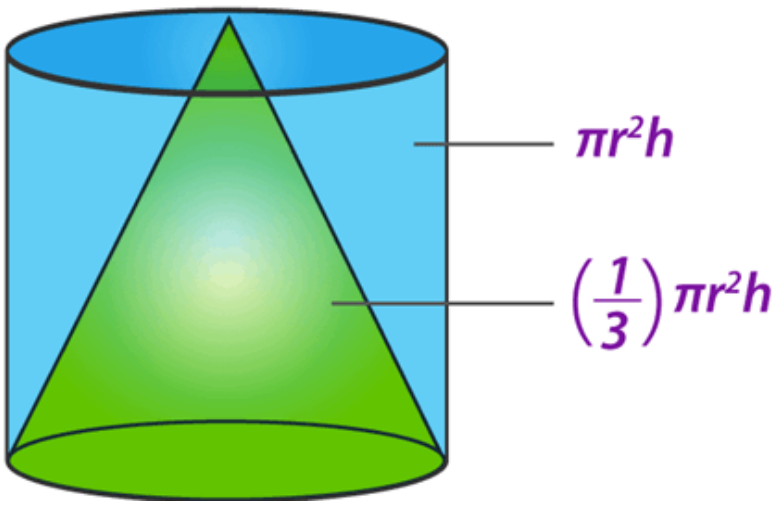
$'h'$ is the height of the cone

As we can see from the above cone formula, the capacity of a cone is one-third of the capacity of the cylinder. That means if we take 1/ 3rd of the volume of the cylinder, we get the formula for cone volume.

Note: The formula for the volume of a regular cone or right circular cone and the oblique cone is same.

Derivation of Cone Volume

Cone as a triangle which is being rotated about one of its vertices. Now, think of a scenario where we need to calculate the amount of water that can be accommodated in a conical flask. In other words, we mean to calculate the capacity of this flask. The capacity of a conical flask is basically equal to the volume of the cone involved. Thus, the volume of a three-dimensional shape is equal to the amount of space occupied by that shape.



Take a cylindrical container and a conical flask of the same height and same base radius. Add water to the conical flask such that it is filled to the brim. Start adding this water to the cylindrical

container you took. Notice it doesn't fill up the container fully. Repeat & you still observe some vacant space in the container. Repeat this experiment once again; you will notice this time the cylindrical container is completely filled. Thus, the **volume of a cone is equal to one-third of the volume of a cylinder having the same base radius and height**.

Now let us derive its formula. Suppose a cone has a circular base with radius $'r'$ and its height is $'h'$. The volume of this cone will be equal to one-third of the product of the area of the base and its height. Therefore,

$V = \frac{1}{3} \times$ Area of Circular Base $\times$ Height of the Cone

Since, we know by the formula of area of the circle, the base of the cone has an area equals to;

$$B = \pi r^2$$

Hence, substituting this value we get;

$$V = \frac{1}{3} \times \pi r^2 \times h$$

where V is the volume, r is the radius and h are the height.

Examples

Q. 1: **Calculate the volume if** $r = 3\,cm$ **and** $h = 6\,cm$.

Solution:

- Here given,

  $$r = 2, h = 5$$

- Using the Volume of Cone formula
- The **volume of a cone** $= \left(\frac{1}{3}\right) \pi r^2 h$ **cubic units**

  $$V = \left(\frac{1}{3}\right) \times 3.14 \times 3 \times 3 \times 6$$

- Here above equation 3 and 3 cancel.

  $$V = 3.14 \times 3 \times 6$$

- Multiplication of $3.14, 3,$ and 6

  $$= 56.52 cm^3$$

- Therefore, the volume of a cone $= 56.52 cm^3$

Q. 2: If the height of a given cone is $12\,cm$ **and the diameter of the circular base is** $8\,cm.$ **Then find its volume**.

Solution:

- Here given the height of cone $= 12\,cm$
- Diameter of the circular base $= 8\,cm.$

- So, radius $= \frac{8}{2} = 4\,cm$
- We have to find out the volume of cone.
- By the formula of cone volume, we know;

$$V = \frac{1}{3}\pi r^2 h$$

- We put the value of radius and height.

$$V = \frac{1}{3}\pi(4)^2 \times 12$$

- Since $\pi = \frac{22}{7}$
- Therefore,

$$V = \frac{1}{3} \times \frac{22}{7} \times (4)^2 \times 12$$

- We divide the 12 by 3

$$V = \frac{22}{7} \times (4)^2 \times 4$$

$$V = \frac{22}{7} \times 16 \times 4$$

- Multiplication of 22,16 and 4.

$$V = \frac{1408}{7}$$

- Take the division,

$$V = 201.14 cm^3$$

- Hence, the volume of cube is $201.14 cm^3$.