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## Properties of Integers: Closure Property and Commutative Property

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There are a few properties of integers which determines its operations. These principles or properties help us to solve many equations. Integers are any positive or negative numbers including zero. The integer properties will help to simplify and solve a series of integers easily.

All properties and identities for addition, subtraction, multiplication and division of numbers are applicable to all the integers. Integers include the set of positive numbers, zero and negative numbers which can be represented with the letter $\mathbf{Z}$.

$$
Z=\ldots-5,-4,-3,-2,-1,0,1,2,3,4,5, \ldots
$$

| Property | Operations on Integers |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Name | Addition | Subtraction | Multiplication | Division* |  |
| Closure | $a+b \in Z$ | $a-b \in Z$ | $a \times b \in Z$ | $a \div b \notin Z$ |  |
| Commutative | $a+b=b+a$ | $a-b \neq b-a$ | $a \times b=b \times a$ | $a \div b \neq b \div a$ |  |
| Associative | $(a+b)+c$ <br> $=a+(b+c)$ | $(a-b)-c$ <br> $\ddagger a-(b-c)$ | $(a \times b) \times c$ <br> $=a \times(b \times c)$ | $(a \div b) \div c$ <br> $\ddagger a \div(b \div c)$ |  |
| Distributive | $a \times(b+c)$ <br> $=a b+a c$ | $a \times(b-c)$ <br> $=a b-a c$ | Not applicable | Not applicable |  |
| where $a, b, c \in Z$ | *b is a non-zero integer |  |  |  |  |

## Properties of Integers

Integers have 5 main properties of operation which are:

- Closure Property
- Associative Property
- Commutative Property
- Distributive Property
- Identity Property

| Integer | Addition | Multiplication | Subtraction | Division |
| :--- | :--- | :--- | :--- | :--- |


| Property |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Commutative property | $x+y=y+x$ | $x \times y=y \times x$ | $x-y \neq y-x$ | $x \div y \neq y \div x$ |
| Associative Property | $x+(y+z)=(x+y)+z$ | $x \times(y \times z)=(x \times y) \times z$ | $(x-y)-z \neq x-(y-z)$ | $(x \div y) \div z \neq x \div(y \div z)$ |
| Identity <br> Property | $x+0=x=0+x$ | $x \times 1=x=1 \times x$ | $x-0=x \neq 0-x$ | $x \div 1=x \neq 1 \div x$ |
| Closure <br> Property | $x+y \in Z$ | $x \times y \in Z$ | $x-y \in Z$ | $x \div y \in Z$ |
| Distributive Property | $\begin{aligned} & x \times(y+z)=x \times \\ & x \times(y-z)=x \times \end{aligned}$ | $\begin{aligned} & +x \times z \\ & -x \times z \end{aligned}$ |  |  |
| Properties of Integers |  |  |  |  |

The explanation of each of the integer properties are given below.

## Property 1: Closure Property

Among the various properties of integers, closure property under addition and subtraction states that the sum or difference of any two integers will always be an integer i.e.. if $x$ and $y$ are any two integers, $\mathrm{x}+\mathrm{y}$ and $\mathrm{x}-\mathrm{y}$ will also be an integer.

Example 1: $3-4=3+(-4)=-1$;

$$
(-5)+8=3,
$$

The results are integers.

Closure property under multiplication states that the product of any two integers will be an integer i.e.. if x and y are any two integers, ${ }_{x y}$ will also be an integer.

Example 2: $6 \times 9=54 ;(-5) \times(3)=-15$, which are integers.
Division of integers doesn't follow the closure property, i.e.. the quotient of any two integers x and y , may or may not be an integer.

Example 3: $(-3) \div(-6)=\frac{-3}{-6}=\frac{1}{2}$, is not an integer.

## Property 2: Commutative Property

The commutative property of addition and multiplication states that the order of terms doesn't matter, the result will be the same. Whether it is addition or multiplication, swapping of terms will not change the sum or product. Suppose, $x$ and $y$ are any two integers, then
$\Rightarrow x+y=y+x$
$\Rightarrow x \times y=y \times x$
Example 4: $4+(-6)=-2=(-6)+4$;

$$
10 \times(-3)=-30=(-3) \times 10
$$

But, subtraction $(x-y \neq y-x)$ and division $(x \div y \neq y \div x)$ are not commutative for integers and whole numbers.

Example 5: $4-(-6)=10 ;(-6)-4=-10$
$\Rightarrow 4-(-6) \neq(-6)-4$
Ex: $10 \div 2=5 ; 2 \div 10=\frac{1}{5}$
$\Rightarrow 10 \div 2 \neq 2 \div 10$

## Property 3: Associative Property

The associative property of addition and multiplication states that the way of grouping of numbers doesn't matter; the result will be same. One can group numbers in any way but the answer will remain same. Parenthesis can be done irrespective of the order of terms. Let $x, y$ and $z$ be any three integers, then
$\Rightarrow x+(y+z)=(x+y)+z$
$\Rightarrow x \times(y \times z)=(x \times y) \times z$
Example 6: $1+(2+(-3))=0=(1+2)+(-3)$;

$$
1 \times(2 \times(-3))=-6=(1 \times 2) \times(-3)
$$

Subtraction of integers is not associative in nature i.e.. $x-(y-z) \neq(x-y)-z$.
Example 7: $1-(2-(-3))=-4 ;(1-2)-(-3)=-2$

$$
1-(2-(-3)) \neq(1-2)-(-3)
$$

## Property 4: Distributive Property

The distributive property explains the distributing ability of operation over another mathematical operation within a bracket. It can be either distributive property of multiplication over addition or distributive property of multiplication over subtraction. Here integers are added or subtracted first and then multiplied or multiply first with each number within the bracket and then added or subtracted. This can be represented for any integers $\mathrm{x}, \mathrm{y}$ and z as:
$\Rightarrow x \times(y+z)=x \times y+x \times z$
$\Rightarrow x \times(y-z)=x \times y-x \times z$
Example 8: $-5(2+1)=-15=(-5 \times 2)+(-5 \times 1)$

## Property 5: Identity Property

Among the various properties of integers, additive identity property states that when any integer is added to zero it will give the same number. Zero is called additive identity. For any integer .,

$$
x+0=x=0+x
$$

The multiplicative identity property for integers says that whenever a number is multiplied by the number 1 it will give the integer itself as the product. Therefore, the integer 1 is called the
multiplicative identity for a number. For any integer x,

$$
x \times 1=x=1 \times x
$$

If any integer multiplied by 0 , the product will be zero:

$$
x \times 0=0=0 \times x
$$

If any integer multiplied by -1 , the product will be opposite of the number:

## FAQs

## What Are the Properties of Integers?

Integers have 5 main properties of operation which are as follows:

- Closure Property
- Associative Property
- Commutative Property
- Distributive Property
- Identity Property

What is the Difference between Commutative and Associative Properties of Integers?
In commutative property, the integers can be rearranged in any way and still the result will be the same. in case of associative property, integers can be grouped in any way using parenthesis and still the result will be the same.

- Commutative Property: $\mathrm{a}+\mathrm{b}=\mathrm{b}+\mathrm{a}$
- Associative Property: $(a+b)+c=a+(b+c)$

What Are the 4 Integer Operations?
The four integer operations are:

- Addition
- Subtraction
- Multiplication
- Division

