Prime Factorization and Division Method for HCF: Least Common Multiple (LCM)

The two important methods which are used to find the LCM (Least Common Multiple) and the HCF (Highest Common Factor) of the numbers are **Prime Factorization Method** and **Division Method**. Both the methods are explained here with many examples. Basically, we determine the prime factors of the given number such as $24, 12, 30, 100$, etc. with this method.

Least Common Multiple (LCM)

The least or smallest common multiple of any two or more given natural numbers are termed as LCM. It is also termed as Lowest Common Multiple.

For example, LCM of $10, 15, \text{and} 20$ is 60 .

Highest Common Factor (HCF)

The largest or greatest factor common to any two or more given natural numbers is termed as HCF of given numbers. It is also known as GCD (Greatest Common Divisor) .

For example, HCF of $4, 6 \text{and} 8 \text{is} 2.$

How to Find LCM and HCF?

We can find HCF and LCM of given natural numbers by two methods i.e.. by prime factorization method or alternatively by division method. In the **prime factorization method**, given numbers are written as the product of prime factors. While in the division method, given numbers are divided by the least common factor and continue still remainder is zero.

Note: *Prime numbers are numbers which have only two factors i.e.. one and the number itself)*

LCM by Prime Factorization Method

Here given natural numbers are written as the product of prime factors. The lowest common multiple will be the product of the all prime factors with the highest degree (power) .

Example 1:

Find the LCM of 20 and 12 by prime factorization method.

Solution:

Step 1: To find LCM of 20 and 12 write each number as a product of prime factors:

$$20 = 2 \times 2 \times 5 = 2^2 \times 5$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

Step 2: Multiply all the prime factors with the highest degree.

Here we have 2 with highest power 2 and other prime factors 3 and 5. Multiply all these to get LCM.

LCM of 20 and 12 $2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5 = 60$

LCM by Division Method

In this method divide the given numbers by common prime number until the remainder is a prime number or one. LCM will be the product obtained by multiplying all divisors and remaining prime numbers.

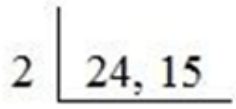
Example 2:

Find the LCM of 24 and 15 by division method.

Solution:

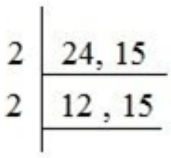
Step 1: Divide the given numbers by the least prime number.

Here, 2 is the least number which will divide 24.



Step 2: Write the quotient and the number which is not divisible by the above prime number in the second row.

In the second row, write the quotient we get after the division of 24 by 2. Since 15 is not divisible by 2 write 15 in the second row as it is.



Step 3: Divide the numbers with another least prime number.

Step 4: Continue division until the remainder is a prime number or 1.

2	24, 15
2	12 , 15
2	6 , 15
3	3 , 15
5	1 , 5
	1 , 1

Step 5: Multiply all the divisors and remaining prime number (if any) to obtain the LCM.

LCM of 24 and 15 $2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5 = 120$

HCF by Prime Factorization Method

Given natural numbers are written as the product of prime factors. To obtain the highest common factor multiply all the common prime factors with the lowest degree (power) .

Example 1:

Find the HCF of 20 and 12 by prime factorization method.

Solution:

Step 1: To find HCF of 20 and 12 write each number as a product of prime factors.

$20 = 2 \times 2 \times 5 = 2^2 \times 5$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

Step 2: Multiply all the common prime factors with the lowest degree.

Here we have only 2 as a common prime factor with the lowest power of 2.

HCF of $20 \text{and} 12 = 4$

HCF by Division Method

In this method divide the largest number by the smallest number of the given numbers until the remainder is zero. The last divisor will be the HCF of given numbers.

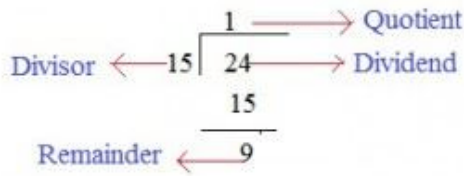
Example 2:

Find the LCM of 24 and 15 by division method.

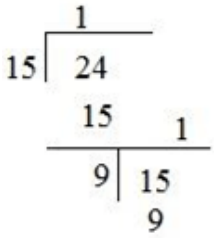
Solution:

Step 1: Divide the largest number by the smallest number.

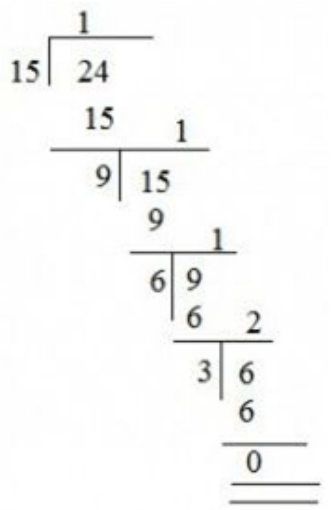
Here, the largest number is 24 and the smaller one is 15. Divide 24 by 15



Step 2: Take divisor as new dividend and remainder as the new divisor, i.e.. divide the first divisor by the first remainder.



Step 3: Proceed this still the remainder is zero and the last divisor will be the HCF of the given numbers.



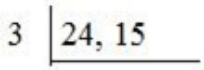
Therefore, HCF of 24and15is3 .

Alternatively, we can divide both the numbers by the least common prime factor still there is no more common prime factor. Multiply all divisors to get the HCF of given numbers.

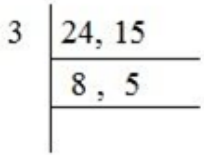
Consider the above example, HCF of 24 and 15 can be calculated by following steps:

Step 1: Divide the given numbers by the least common prime factor.

Here, 3 is the least common prime factor of 24 and 15.



Step 2: Continue still there is no more common prime factor. Then multiply all the divisor.



Division of 24 and 15 by 3 will leave 8 and 5 as remainders, respectively. 8 and 5 do not have a common prime factor.

Hence, the HCF of 24 and 15 is 3.

Example: 3 Seema, Meena and Reema begin to jog around a circular stadium and they complete their revolutions in 54 seconds, 42 seconds and 63 seconds, respectively. After how much time will they come together at the starting point?

Solution:

L. C. M of $54, 42 \text{and} 63$

$54 = 2 \times 3 \times 3 \times 3$

$42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7$

LCM of 54,42 and 63 $= 3 \times 3 \times 3 \times 2 \times 7 = 378$

So, the three girls will come together at the starting point in 378 seconds $= 6 \min 18 \text{sec}$