Permutation and Combination: Permutation and Combination Formulas

Permutation and combination are all about counting and arrangements made from a certain group of data.

What is Permutation?

In mathematics, **permutation relates to the act of arranging all the members of a set into some sequence or order**, or if the set is already ordered, rearranging its elements, a process called permuting. Permutations occur, in more or less prominent ways, in almost every area of mathematics. They often arise when different orderings on certain finite sets are considered.

What is Combination?

The **combination is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter**. In smaller cases, it is possible to count the number of combinations. Combination refers to the combination of n things taken k at a time without repetition. To refer to combinations in which repetition is allowed, the terms k-selection or k-combination with repetition are often used. Permutation and Combination

Permutation and Combination Formulas

There are many formulas involved in permutation and combination concept. The two key formulas are:

Permutation Formula

A permutation is the choice of r things from a set of n things without replacement and where the order matters.

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Combination Formula

A combination is the choice of r things from a set of n things without replacement and where order does not matter.

$$^{n}C_{r} = \binom{n}{r} = \frac{^{n}P_{r}}{r!} = \frac{n!}{r!\,(n-r)!}$$

Difference between Permutation and Combination

Permutation	Combination
Arranging people, digits, numbers, alphabets, letters, and colours	Selection of menu, food, clothes, subjects, team.

Picking a team captain, pitcher, and shortstop from a group.	Picking three team members from a group.
Picking two favorite colours, in order, from a colour brochure.	Picking two colours from a colour brochure.
Picking first, second and third place winners.	Picking three winners.

Difference between Permutation and Combination

Uses of Permutation and Combination

A permutation is used for list of data (where the order of the data matters) and the combination is used for a group of data (where the order of data doesn't matter) .

Permutation and Combination Examples

Example 1: Find the number of permutations and combinations if $n = 14$ and $r = 5$?

Solution: Given,

$$n = 14 r = 5$$

Using the formula given above:

Permutation:

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$= \frac{14!}{(14-5)!}$$

$$= \frac{14!}{9!}$$

$$= \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9!}{9!}$$

$$= 14 \times 13 \times 12 \times 11 \times 10$$

$$= 2,40,240$$

Combination:

$$^{n}C_{r} = \binom{n}{r} = \frac{^{n}P_{r}}{r!} = \frac{n!}{r!\,(n-r)!}$$

$$= \frac{14!}{5!\,(14-5)!}$$

$$= \frac{14!}{5!9!}$$

$$= \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9!}{5! \times 9!}$$

$$= \frac{14 \times 13 \times 12 \times 11 \times 10}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 14 \times 13 \times 11$$

$= 2002$

Example 2: **In a dictionary, If all permutations of the letters of the word AGAIN are arranged in an order. What is the 49^{th} word?**

Solution:

Start with the letter A	The arranging the other 4 letters: G, A, I, N $= 4! = 4 \times 3 \times 2 \times 1 = 24$	First 24 words
Start with the letter G	Arrange A, A, I and N in different ways: $\frac{4!}{2!1!1!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 4 \times 3 = 12$	Next 12 words
Start with the letter I	Arrange A, A, G and N in different ways: $\frac{4!}{2!1!1!} = 12$	Next 12 words

In a Dictionary, if All Permutations of the Letters of the Word AGAIN Are Arranged in an Order. What is the 49^{th} Word?

This accounts up to the 48^{th} word. The 49^{th} word is “NAAGI” .

Example 3: In how many ways a committee consisting of 5 men and 3 women, can be chosen from 9 men and 12 women.

Solution:

Choose 5 men out of 9 men $= {}^{9}C_5 ways = \frac{n!}{r!\,(n-r)!}$

$= \frac{9!}{5!\,(9-5)!}$

$= \frac{9!}{5!4!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 4 \times 3 \times 2 \times 1}$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$

$= 9 \times 7 \times 3$

$= 126$

Choose 3 women out of 12 women = ${}^{12}C_3$ ways

$= \frac{12!}{3!\,(12-3)!}$

$= \frac{12!}{3!9!}$

$= \frac{12 \times 11 \times 10 \times 9!}{3! \times 9!}$

$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1}$

$= 2 \times 11 \times 10$

$= 220\ ways$

The committee can be chosen in $220 \times 126 = 27720$ ways.