Joint Probability: Formula for Joint Probability and Joint Probability Distribution

Probability is a branch of mathematics which deals with the occurrence of a random event. In simple words it is the likelihood of a certain event. A statistical measure that calculates the likelihood of two events occurring together and at the same point in time is called Joint probability.

Let A and B be the two events, joint probability is the probability of event B occurring at the same time that event A occurs.

Formula for Joint Probability

Notation to represent the joint probability can take a few different forms. The following formula represents the joint probability of events with intersection.

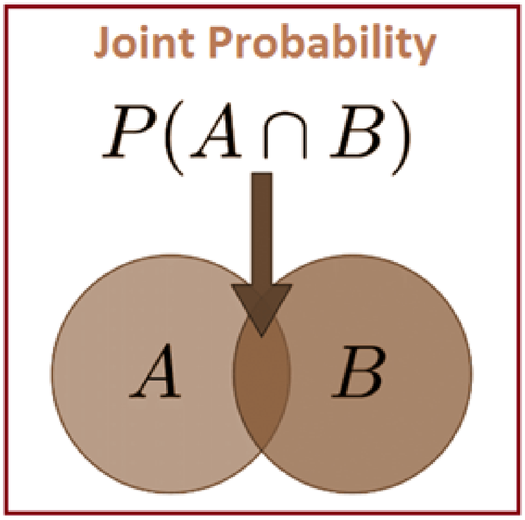
$$P\left(A \bigcap B\right)$$

where,

$A, B =$ Two events

$P(\text{AandB}), P(AB) =$ The joint probability of A and B

The symbol $''\cap''$ in a joint probability is called an intersection. The probability of event A and event B happening is the same thing as the point where A and B intersect. Hence, the joint probability is also called the intersection of two or more events. We can represent this relation using a Venn diagram as shown below.



Joint Probability Distribution

Let $A, B, \dots,$ be the random variables which are defined on a probability space. The probability distribution that gives the probability that each of $A, B, \dots$ falls in any particular range or discrete set of values specified for that variable is defined as the joint probability distribution for $A, B, \dots$ In the case of only two random variables, this is called a bivariate distribution, otherwise it is a multivariate distribution.

The joint probability distribution can be expressed in different ways based on the nature of the variable. In case of discrete variables, we can represent a joint probability mass function. For continuous variables it can be represented as a joint cumulative distribution function or in terms of a joint probability density function.

Joint Probability Examples

Let us see some examples of how to find the joint probability with solutions.

Example: Find the probability that the number three will occur twice when two dice are rolled at the same time.

Solution:

Number of possible outcome when a die is rolled = 6

i.e.. $1, 2, 3, 4, 5, 6$

Let A be the event of occurring 3 on first die and B be the event of occurring 3 on the second die.

Both the dice have six possible outcomes, the probability of a three occurring on each die is $\frac{1}{6}$ $P(A) = \frac{1}{6}$

$$P(B) = \frac{1}{6}$$

We have to find out the probability, so we multiplied the both event's probability.

$$P(A, B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Joint Probability Table

A joint probability distribution represents a probability distribution for two or more random variables. Instead of events being labeled A and B, the condition is to use X and Y as given below.

$$f(x, y) = P(X = x, Y = y)$$

The main purpose of this is to look for a relationship between two variables. For example, the below table shows some probabilities for events X and Y happening at the same time:

This table can be used to find the probabilities of events.

X

Y		1	2	3
	1	$\frac{1}{6}$	0	$\frac{1}{6}$
	2	$\frac{1}{6}$	$\frac{1}{6}$	0
	3	0	$\frac{1}{6}$	$\frac{1}{6}$

Example: Find the probability of $X = 3$ and $Y = 3$.

Solution: From the above table, identify the probability of $X = 3$ and $Y = 3$.

That is $\frac{1}{6}$.

X

Y		1	2	3
	1	$\frac{1}{6}$	0	$\frac{1}{6}$
	2	$\frac{1}{6}$	$\frac{1}{6}$	0
	3	0	$\frac{1}{6}$	$\boxed{\frac{1}{6}}$

Example 1: What is the joint probability of rolling the number five twice in a fair six-sided dice?

Solution:

Event $''A'' =$ The probability of rolling a 5 in the first roll is $\frac{1}{6} = 0.1666.$

Event $''B'' =$ The probability of rolling a 5 in the second roll is $\frac{1}{6} = 0.1666.$

Therefore, the joint probability of event "A" and "B" is
$P\left(\frac{1}{6}\right) \times P\left(\frac{1}{6}\right) = 0.1666 \times 0.1666 = 0.02777 = 2.8\%.$

Example 2: What is the joint probability of getting a head followed by a tail in a coin toss?

Solution:

Event "A" = The probability of getting a head in the first coin toss is $\frac{1}{2} = 0.5.$

Event "B" = The probability of getting a tail in the second coin toss is $\frac{1}{2} = 0.5.$

Therefore, the joint probability of event “A” and “B” is $P\left(\frac{1}{2}\right) \times P\left(\frac{1}{2}\right) = 0.5 \times 0.5 = 0.25 = 25\%.$

Example 3: What is the joint probability of drawing a number ten card that is black?

Solution:

Event $''A''=$ The probability of drawing a $10 = \frac{4}{52} = 0.0769$ (In total card, card of ten is 4 cards)

Event $''B''=$ The probability of drawing a black card $= \frac{26}{52} = 0.50$ (In total card, black card is total 26) .

Therefore, the joint probability of event “A” and “B” is
$P\left(\frac{4}{52}\right) \times P\left(\frac{26}{52}\right) = 0.076 \times 0.50 = 0.0385 = 3.9\%.$