Inverse Trigonometric Functions: Properties of Inverse Part 1

What is Inverse Trigonometric Function?

Considering the Domain and Range of the Inverse Functions, Following Formulas Are Important to Be Noted:

$$\sin\left(\sin^{-1} x\right) = x, \text{if} - 1 \leqslant x \leqslant 1$$

$$\cos\left(\cos^{-1} x\right) = x, \text{if} - 1 \leqslant x \leqslant 1$$

$$\tan\left(\tan^{-1} x\right) = x, \text{if} - \infty \leqslant x \leqslant \infty$$

$$\cos\left(\cos^{-1} x\right) = x, \text{if} - \infty \leqslant x \leqslant \infty$$

$$\cot\left(\cot^{-1} x\right) = x, \text{if} - \infty \leqslant x \leqslant \infty$$

$$\sec\left(\sec^{-1} x\right) = x, \text{if} - \infty \leqslant x \leqslant -1 \text{or} 1 \leqslant x \leqslant \infty$$

$$co\sec\left(\cos ec^{-1} x\right) = x, \text{if} - \infty \leqslant x \leqslant -1 \text{or} 1 \leqslant x \leqslant \infty$$

- The inverse trigonometric functions are an important aspect of trigonometric functions, included in the syllabus for class 12 students.
- In mathematics, the inverse trigonometric functions are the inverse functions of the trigonometric functions.
- Inverses of trigonometric functions exist solely due to the restrictions existing on the domains and their respective ranges.
- Being able to solve inverse trigonometric function problems starts by understanding the trigonometric ratios first.
- Specifically, they are the inverses of the sine, cosine, tangent, cotangent, secant, and cosecant functions, and are used to obtain an angle from any of the angle's trigonometric ratios
- The behavior of these trigonometric functions is usually represented in the form of graphical methods.
- They play an essential role in calculus as they help to define different integrals.
- Major applications of inverse trigonometric functions in everyday life are in the fields of science and engineering.

Properties of Inverse Trigonometric Functions

- The properties of Inverse Trigonometric Functions help to prove a distinct relationship between the different trigonometric entities such as $\sin x, \cos x, \tan x, \operatorname{cosec} x, \sec x$, and $\cot x$.

- There are a few inverse trigonometric functions properties which are crucial to not only solve problems but also to have a deeper understanding of this concept.
- The domain of a function is defined as the set of every possible independent variable where the function exists. Inverse Trigonometric Functions are defined in a certain interval.
- The results obtained with the help of these properties are valid within the principal branches of the inverse trigonometric functions.
- To recall, inverse trigonometric functions are also called "Arc Functions." For a given value of a trigonometric function; they produce the length of arc needed to obtain that value.
- These properties are valid for some values of , where these inverse trigonometric functions are defined with.
- The range of an inverse function is defined as the range of values of the inverse function that can attain with the defined domain of the function.

Property Set 1

$$\sin^{-1}(x) = \cos ec^{-1}\left(\frac{1}{x}\right), \times \in [-1,1] - \{0\}$$

$$\cos^{-1}(x) = \sec^{-1}\left(\frac{1}{x}\right), \times \in [-1,1] - \{0\}$$

$$\tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right), \text{if} x > 0\,(\text{or})\cot^{-1}\left(\frac{1}{x}\right) - \pi, \text{if} x < 0$$

$$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right), \text{if} x > 0\,(\text{or})\tan^{-1}\left(\frac{1}{x}\right) + \pi, \text{if} x < 0$$

Property Set 2

$$\sin^{-1}(-x) = -\sin^{-1}(x)$$

$$\tan^{-1}(-x) = -\tan^{-1}(x)$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$$

$$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

Derivatives of Inverse Trigonometric Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, x \neq \pm 1$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, x \neq \pm 1$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cot^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}, x \neq \pm 1, 0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(csc^{-1}x\right) = \frac{-1}{|x|\sqrt{x^2-1}}, x \neq \pm 1, 0$$