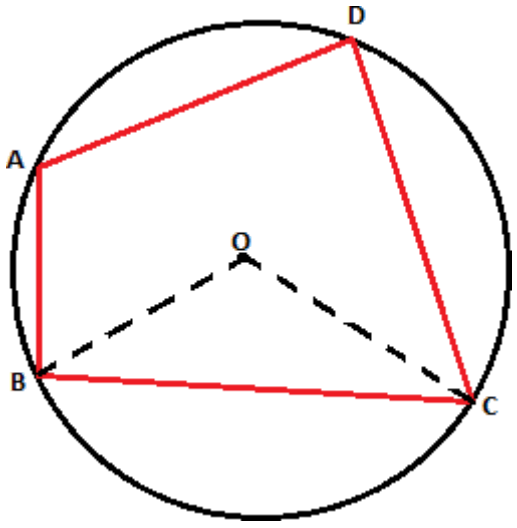
FlexiPrep

Cyclic Quadrilateral: Cyclic Quadrilateral Theorem and Properties of Cyclic Quadrilateral Theorem (For CBSE, ICSE, IAS, NET, NRA 2022)

A quadrilateral is a 4-sided polygon bounded by 4 finite line segments. The word 'quadrilateral' is composed of two Latin words, *Quadric* meaning 'four' and *latus* meaning 'side' . It is a two-dimensional figure having four sides (or edges) and four vertices. A circle is the locus of all points in a plane which are equidistant from a fixed point. If all the four vertices of a quadrilateral ABCD lie on the circumference of the circle, then ABCD is a cyclic quadrilateral. In other words, if any four points on the circumference of a circle are joined, they form vertices of a **cyclic quadrilateral**. It can be visualized as a quadrilateral which is inscribed in a circle, i.e.. all four vertices of the quadrilateral lie on the circumference of the circle.

What is a Cyclic Quadrilateral?

In the figure given below, the quadrilateral ABCD is cyclic.



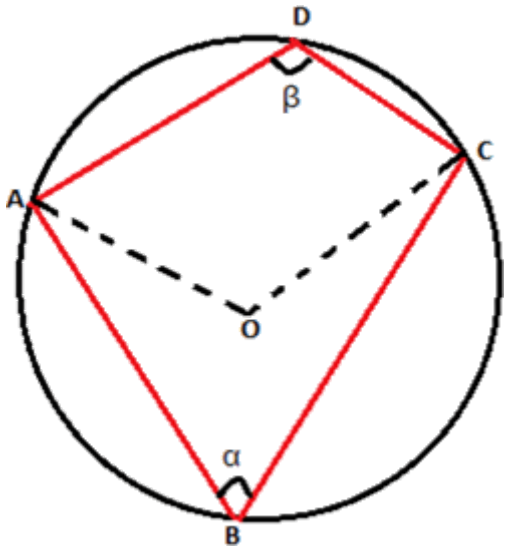
Let us do an activity. Take a circle and choose any 4 points on the circumference of the circle. Join these points to form a quadrilateral. Now measure the angles formed at the vertices of the cyclic quadrilateral. The sum of the angles formed at the vertices is always 360° and the sum of angles formed at the opposite vertices is always supplementary. This property can be stated as a theorem as:

Cyclic Quadrilateral Theorem

Theorem 1: In a cyclic quadrilateral, the sum of either pair of opposite angles is supplementary.

Proof: Let us now try to prove this theorem.

Given: A cyclic quadrilateral ABCD inscribed in a circle with center O. Construction: Join the vertices A and C with center O.



The converse of this theorem is also true which states that if opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.

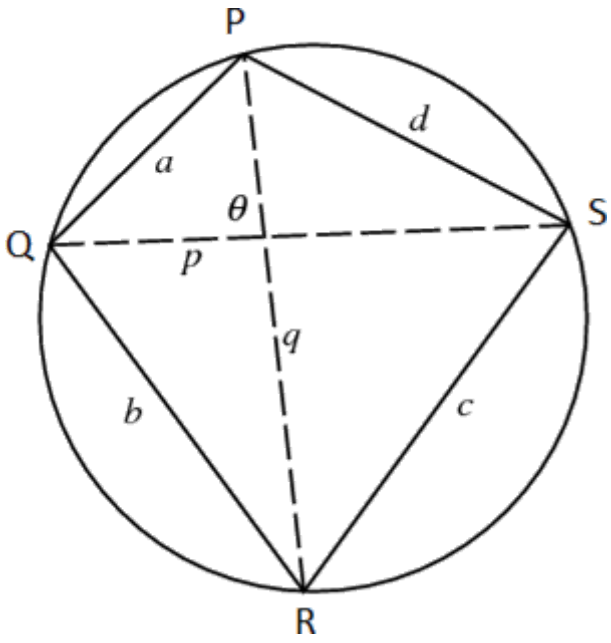
S. NO	Statement	Reason

1.	$\angle ADC = 2\angle ABC = 2\alpha$	Theorem: Angle subtended by same arc is half of the angle subtended at the center.
2.	Reflex $\angle ADC = 2\angle ADC = 2\beta$	Theorem: Angle subtended by same arc is half of the angle subtended at the center.
3.	$\angle ADC + \text{reflex } \angle ADC = 360°$ $2\angle ABC + 2\angle ADC = 360°$ $2\alpha + 2\beta = 360°$ $\alpha + \beta = 180°$	Using statement 1 and 2.
The Converse of this Theorem is Also True Which States That if Opposite Angles of a Quadrilateral Are Supplementary then the Quadrilateral is Cyclic		

Theorem 2: The ratio between the diagonals and the sides can be defined and is known as Cyclic quadrilateral theorem. If there's a quadrilateral which is inscribed in a circle, then the product of the diagonals is equal to the sum of the product of its two pairs of opposite sides.

If PQRS is a cyclic quadrilateral, PQ and ₹ , and QR and PS are opposite sides. PR and QS are the diagonals.

$$(PQ \times ₹) + (QR \times PS) = PR \times QS$$



Properties of Cyclic Quadrilateral

In a cyclic quadrilateral, the sum of a pair of opposite angles is $180°$ (supplementary)
If the sum of two opposite angles are supplementary, then it's a cyclic quadrilateral
The area of a cyclic quadrilateral is $[s\,(s-a)\,(s-b)\,(s-c)\,(s-c)]\,0.5$ where a, b, c, and d, are the four sides of the quadrilateral and the perimeter is $2s$
The four vertices of a cyclic quadrilateral lie on the circumference of the circle
To get a rectangle or a parallelogram, just join the midpoints of the four sides in order
If PQRS is a cyclic quadrilateral, then $\angle SPR = \angle SQR, \angle QPR = \angle QSR, \angle PQS = \angle PRS, \angle QRP = \angle QSP$.
If T is the point of intersection of the two diagonals, $PT \times TR = QT \times TS$

The exterior angle formed if any one side of the cyclic quadrilateral is produced is equal to the interior angle opposite to it
In a given cyclic quadrilateral, d_1/d_2 = sum of the product of opposite sides, which shares the diagonals endpoints
If it is cyclic quadrilateral, then the perpendicular bisectors will be concurrent compulsorily
In a cyclic quadrilateral, the four perpendicular bisectors of the given four sides meet at the center O

Properties of Cyclic Quadrilateral

Cyclic Quadrilateral Examples

Question: Find the value of angle D of a cyclic quadrilateral, if angle B is 70°.

Solution:

As ABCD is a cyclic quadrilateral, so the sum of a pair of two opposite angles will be 180°.

$$\angle B + \angle D = 180°$$

$$70° + \angle D = 180°$$

$$\angle D = 180° - 70°$$

$$\angle D = 110°$$

The value of angle D is 110°.

Question: Find the value of $\angle D$ of a cyclic quadrilateral, if $\angle B$ is 50°.

Solution:

As ABCD is a cyclic quadrilateral, so the sum of a pair of two opposite angles will be 180°.

$$\angle B + \angle D = 180°$$

Put the value of $\angle B$

$$50° + \angle D = 180°$$

80° take the opposite side.

$$\angle D = 180° - 50°$$

$$\angle D = 130°$$

The value of angle D is 130°.

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