

FlexiPrep**NCERT Class 11 Mathematics Solutions: Chapter 9 – Sequences and Series Miscellaneous Exercise 9 Part 13 (For CBSE, ICSE, IAS, NET, NRA 2022)**

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Arithmetic Series

An arithmetic series is the sum of an arithmetic sequence.

Formulas for Arithmetic Series:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

where

a_1 is the first term

a_n is the n^{th} term

n is the number of terms

d is the common difference

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1. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9S_2^2 = S_3(1 + 8S_1)$

Answer:

From the given information,

$$S_1 = \frac{n(n+1)}{2}$$

$$S_3 = \frac{n^2(n+1)^2}{4}$$

So,

$$\begin{aligned} S_3(1 + 8S_1) &= \frac{n^2(n+1)^2}{4} \left[1 + \frac{8n(n+1)}{2} \right] \\ &= \frac{n^2(n+1)^2}{4} [1 + 4n^2 + 4n] \\ &= \frac{n^2(n+1)^2}{4} (2n+1)^2 \\ &= \frac{[n(n+1)(2n+1)]^2}{4} \dots \text{eq (1)} \end{aligned}$$

Also,

$$\begin{aligned} 9S_2^2 &= 9 \frac{[n(n+1)(2n+1)]^2}{(6)^2} \\ &= \frac{9}{36} [n(n+1)(2n+1)]^2 \\ &= \frac{[n(n+1)(2n+1)]^2}{4} \dots \text{eq (2)} \end{aligned}$$

So, from eq (1) and (2)

$$9S_2^2 = S_3(1 + 8S_1)$$

2. Find the sum of the following series up to n terms: $\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \dots$

Answer:

The n^{th} term of the given series is $\frac{1^3 + 2^3 + 3^3 + \dots + n^3}{1 + 3 + 5 + \dots + (2n-1)} = \frac{\left[\frac{n(n+1)}{2}\right]^2}{1 + 3 + 5 + \dots + (2n-1)}$

Here, $1, 3, 5, \dots, (2n-1)$ is an A. P. with first term a , last term $(2n-1)$ and number of terms as n .

$$\therefore 1 + 3 + 5 + \dots + (2n-1) = \frac{n}{2} [2 \times 1 + (n-1)2] = n^2$$

$$\therefore a_n = \frac{n^2(n+1)^2}{4n^2} = \frac{(n+1)^2}{4} = \frac{1}{4}n^2 + \frac{1}{2}n + \frac{1}{4}$$

$$\begin{aligned} \therefore S_n &= \sum_{k=1}^n a_k = \sum_{k=1}^n \left(\frac{1}{4}k^2 + \frac{1}{2}k + \frac{1}{4} \right) \\ &= \frac{1}{4} \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \frac{n(n+1)}{2} + \frac{1}{4}n \\ &= \frac{n[(n+1)(2n+1) + 6(n+1) + 6]}{24} \end{aligned}$$

$$\begin{aligned} &= \frac{n [2n^2 + 3n + 1 + 6n + 6 + 6]}{24} \\ &= \frac{n (2n^2 + 9n + 13)}{24} \end{aligned}$$

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