FlexiPrep

NCERT Class 11 Mathematics Solutions: Chapter 9 – Sequences and Series Miscellaneous Exercise 9 Part 13 (For CBSE, ICSE, IAS, NET, NRA 2022)

Arithmetic Series

An arithmetic series is the sum of an arithmetic sequence.

Formulas for Arithmetic Series:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

where

a_1 is the first term

a_n is the n^{th} term

n is the number of terms

1. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9S_2^2 = S_3\left(1 + 8S_1\right)$

Answer:

From the given information,

$$S_1 = \frac{n(n+1)}{2}$$

$$S_3 = \frac{n^2(n+1)^2}{4}$$

So,

$$S_3\,(1 + 8S_1) = \frac{n^2(n+1)^2}{4}\left[1 + \frac{8n(n+1)}{2}\right]$$

$$= \frac{n^2(n+1)^2}{4}\left[1 + 4n^2 + 4n\right]$$

$$= \frac{n^2(n+1)^2}{4}(2n+1)^2$$

$$= \frac{[n(n+1)(2n+1)]^2}{4} \text{ ... eq (1)}$$

Also,

$$9S_2^2 = 9\frac{[n\,(n+1)\,(2n+1)]^2}{(6)^2}$$

$$= \frac{9}{36}[n\,(n+1)\,(2n+)]^2$$

$$= \frac{[n\,(n+1)\,(2n+1)]^2}{4} \text{ ... eq (2)}$$

So, from eq (1) and (2)

$$9S_2^2 = S_3\left(1 + 8S_1\right)$$

2. Find the sum of the following series up to n terms: $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$

Answer:

The n^{th} term of the given series is $\frac{1^3+2^3+3^3+\dots+n^3}{1+3+5+\dots+(2n-1)} = \frac{\left[\frac{n(n+1)}{2}\right]^2}{1+3+5+\dots+(2n-1)}$

Here, 1,3,5, $\dots$ $(2n-1)$ is an A. P. with first term a, last term $(2n-1)$ and number of terms as n.

$$\therefore 1+3+5+\dots+(2n-1) = \frac{n}{2}[2\times 1 + (n-1)\,2] = n^2$$

$$\therefore a_n = \frac{n^2(n+1)^2}{4n^2} = \frac{(n+1)^2}{4} = \frac{1}{4}n^2 + \frac{1}{2}n + \frac{1}{4}$$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n}\left(\frac{1}{4}k^2 + \frac{1}{2}k + \frac{1}{4}\right)$$

$$= \frac{1}{4}\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\frac{n(n+1)}{2} + \frac{1}{4}n$$

$$= \frac{n\,[(n+1)\,(2n+1) + 6\,(n+1) + 6]}{24}$$

$$= \frac{n\left[2n^2 + 3n + 1 + 6n + 6 + 6\right]}{24}$$

$$= \frac{n\left(2n^2 + 9n + 13\right)}{24}$$