NCERT Class 11 Mathematics Solutions: Chapter 9 – Sequences and Series Miscellaneous Exercise 9 Part 1

Arithmetic Series

An arithmetic series is the sum of an arithmetic sequence.

Formulas for Arithmetic Series:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

where

a_1 is the first term

a_n is the n^{th} term

n is the number of terms

d is the common difference

1. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A. P. is equal to twice the m^{th} term.

Answer:

Consider a and d be the first term and the common difference of the A. P. respectively.

It is known that the kth term of an A. P. is given by

$a^k = a + (k-1)d$

$\therefore a_{m+n} = a + (m+n-1)d$

$a_{m-n} = a + (m-n-1)d$

$a_m = a + (m-1)d$

$\therefore a_{m+n} + a_{m-n} = a + (m+n-1)d + a + (m-n-1)d$

$= 2a + (m+n-1+m-n-1)d$

$= 2a + (2m-2)d$

$= 2a + 2(m-1)d$

$= 2[a + (m-1)d]$

$= 2a_m$

So, the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A. P. is equal to twice the m^{th} term.

2. If the sum of three numbers in A. P., is 24 and their product is 440, find the numbers.

Answer:

Consider the three numbers in A. P. be $a-d, a,$ and $a+d$.

According to the given information,

$(a-d)+(a)+(a+d)=24$ ... (1)

$\rightarrow 3a=24$

$\therefore a=8$

$(a-d)\,a\,(a+d)=440$ ... (2)

$\therefore (8-d)(8)(8+d)=440$

$\therefore (8-d)(8+d)=55$

$\therefore 64-d^2=55$

$\therefore d^2=64-55=9$

$\therefore d=\pm 3$

So, when $d=3$, the numbers are $5, 8, \text{and} 11$ and when $d=-3$,

The numbers are $11, 8, \text{and} 5$.

So, the three numbers are $5, 8, \text{and} 11$.