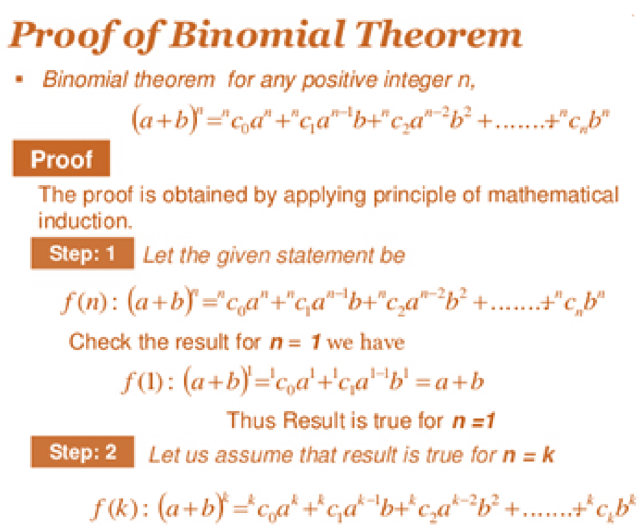
NCERT Class 11 Mathematics Solutions: Chapter 8 – Binomial Theorem Miscellaneous Exercise Part 1



1. Find $a, b \text{and} n$ in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, $7290 \text{and} 30375$, respectively.

Answer:

It is known that $(r+1)^{th} term, (T_{r+1})$, in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

The first three terms of the expansion are given as $729, 7290, \text{and} 30375$ respectively

So,

$T_1 = {}^nC_0 a^{n-0} b^0 = a^n = 729$ … eq (1)

$$T_2 = {}^nC_1 a^{n-1} b^1 = na^{n-1} b = 7290 \quad \text{... eq (2)}$$

$$T_3 = {}^nC_2 a^{n-2} b^2 = \frac{n(n-1)}{2} a^{n-2} b^2 = 30375 \quad \text{... eq (3)}$$

Dividing eq (2) by (1)

$$\frac{na^{n-1} b}{a^n} = \frac{7290}{729}$$

$$\Rightarrow \frac{nb}{a} = 10 \quad \text{... eq (4)}$$

Dividing eq (3) by (2)

$$\therefore \frac{n(n-1) a^{n-2} b^2}{2na^{n-1} b} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1) b}{2a} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1) b}{a} = \frac{30375 \times 2}{7290} = \frac{25}{3}$$

$$\Rightarrow \frac{nb}{a} - \frac{b}{a} = \frac{25}{3}$$

$$\Rightarrow 10 - \frac{b}{a} = \frac{25}{3} \quad \text{[Using (4)]}$$

$$\Rightarrow \frac{b}{a} = 10 - \frac{25}{3} = \frac{5}{3} \quad \text{... eq (5)}$$

From eq (4) and (5)

$$n \bullet \frac{5}{3} = 10$$

$$\Rightarrow n = 6$$

Substituting $n = 6$ in eq (1)

$$a^6 = 729$$

$$\Rightarrow a = \sqrt[6]{729} = 3$$

From eq (5)

$$\frac{b}{3} = \frac{5}{3}$$

$$\Rightarrow b = 5$$

So, $a = 3, b = 5 \text{and} n = 6$.