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NCERT Class 11 Mathematics Solutions: Chapter 8 – Binomial Theorem Miscellaneous Exercise Part 1

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Proof of Binomial Theorem

- Binomial theorem for any positive integer n ,

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n b^n$$

Proof

The proof is obtained by applying principle of mathematical induction.

Step: 1 Let the given statement be

$$f(n): (a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n b^n$$

Check the result for $n = 1$ we have

$$f(1): (a+b)^1 = {}^1C_0 a^1 + {}^1C_1 a^{1-1} b^1 = a+b$$

Thus Result is true for $n = 1$

Step: 2 Let us assume that result is true for $n = k$

$$f(k): (a+b)^k = {}^kC_0 a^k + {}^kC_1 a^{k-1} b + {}^kC_2 a^{k-2} b^2 + \dots + {}^kC_k b^k$$

1. Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

Answer:

It is known that $(r+1)^{th} \text{ term}, (T_{r+1})$, in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

The first three terms of the expansion are given as 729, 7290, and 30375 respectively

So,

$$T_1 = {}^nC_0 a^{n-0} b^0 = a^n = 729 \dots \text{eq (1)}$$

$$T_2 = {}^nC_1 a^{n-1} b^1 = na^{n-1} b = 7290 \dots \text{eq (2)}$$

$$T_3 = {}^nC_2 a^{n-2} b^2 = \frac{n(n-1)}{2} a^{n-2} b^2 = 30375 \dots \text{eq (3)}$$

Dividing eq (2) by (1)

$$\frac{na^{n-1}b}{a^n} = \frac{7290}{729}$$

$$\Rightarrow \frac{nb}{a} = 10 \dots \text{eq (4)}$$

Dividing eq (3) by (2)

$$\therefore \frac{n(n-1)a^{n-2}b^2}{2na^{n-1}b} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1)b}{2a} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1)b}{a} = \frac{30375 \times 2}{7290} = \frac{25}{3}$$

$$\Rightarrow \frac{nb}{a} - \frac{b}{a} = \frac{25}{3}$$

$$\Rightarrow 10 - \frac{b}{a} = \frac{25}{3} \text{ [Using (4)]}$$

$$\Rightarrow \frac{b}{a} = 10 - \frac{25}{3} = \frac{5}{3} \dots \text{eq (5)}$$

From eq (4) and (5)

$$n \cdot \frac{5}{3} = 10$$

$$\Rightarrow n = 6$$

Substituting $n = 6$ in eq (1)

$$a^6 = 729$$

$$\Rightarrow a = \sqrt[6]{729} = 3$$

From eq (5)

$$\frac{b}{3} = \frac{5}{3}$$

$$\Rightarrow b = 5$$

So, $a = 3, b = 5$ and $n = 6$.