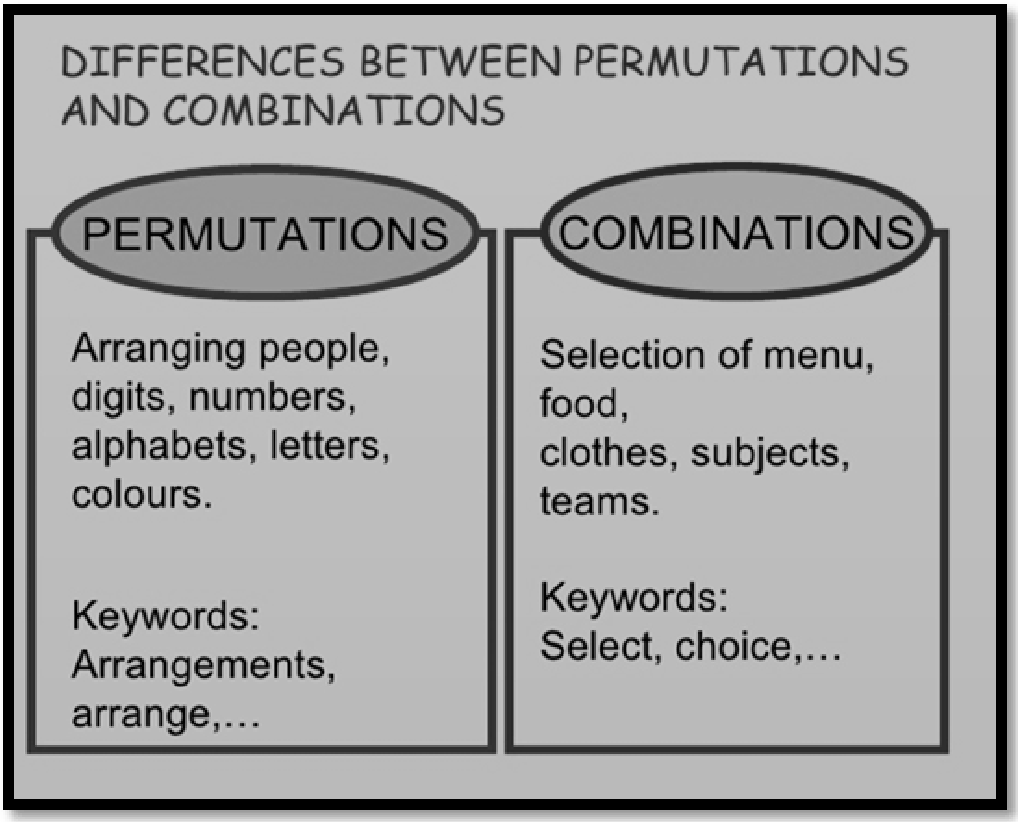
NCERT Class 11 Mathematics Solutions: Chapter 7 – Permutation and Combinations Miscellaneous Exercise Part 1



1. How many words, with or without meaning, each of vowels and consonants can be formed from the letters of the word DAUGHTER?

Answer:

In the word DAUGHTER , there are vowels namely, $A, U, \text{and} E, \text{and} 5$ consonants namely, $D, G, H, T, \text{and} R$.

Number of ways of selecting vowels out of vowels $= {}^3C_2 = 3$

Number of ways of selecting consonants out of consonants $= {}^5C_3 = 10$

So, number of combinations of vowels and consonants $= 3 \times 10 = 30$

Each of these 30 combinations of vowels and consonants can be arranged among themselves in ! ways.

Hence, required number of different words $= 30 \times 5! = 3600$

2. How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Answer:

In the word EQUATION , there are vowels, namely $, A, E, I, O, \text{and} U,$ and consonants, namely, $Q, T, \text{and} N$.

Since all the vowels and consonants have to occur together, both (AEIOU) and (QTN) can be assumed as single objects.

Then, the permutations of these objects taken all at a time are counted.

Corresponding to each of these permutations, there are $5!$ permutations of the five vowels taken all at a time and $3!$ permutations of the consonants taken all at a time.

So, by multiplication principle, required number of words $= 2! \times 5! \times 3! = 1440$

3. A committee of has to be formed from boys and girls. In how many ways can this be done when the committee consists of:

(i) exactly girls?

(ii) at least girls?

(iii) at most girls?

Answer: (i)

A committee of has to be formed from boys and girls.

Since exactly girls are to be there in every committee, each committee must consist of $(7-3) = 4$ boys only.

So, in this case required number of ways $= {}^4C_3 \times {}^9C_4$

$$= \frac{4!}{3!1!} \times \frac{9!}{4!5!}$$

$$= 4 \times \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!}$$

$$= 504$$

Answer: (ii)

Since at least girls are to be there in every committee, the committee can consist of

(a) girls and boys or

(b) girls and boys

girls and boys can be selected in ${}^{4}C_{3} \times {}^{9}C_{4}$ ways.

girls and boys can be selected in ${}^{4}C_{4} \times {}^{9}C_{3}$ ways

So, in this case, required number of ways $= {}^{4}C_{3} \times {}^{9}C_{4} + {}^{4}C_{4} \times {}^{9}C_{3}$

$= 504 + 84$

$= 588$

Answer: (iii)

Since at most girls are to be there in every committee, the committee can consist of

(a) girls and boys

(b) girls and boys

(c) girl and boys

(d) No girl and boys

girls and boys can be selected in ${}^{4}C_{3} \times {}^{9}C_{4}$ ways.

girls and boys can be selected in ${}^{4}C_{2} \times {}^{9}C_{5}$ ways.

girl and boys can be selected in ${}^{4}C_{1} \times {}^{9}C_{6}$ ways.

No girl and boys can be selected in ${}^{4}C_{0} \times {}^{9}C_{7}$ ways.

So, in this case, required number of ways

$$= {}^{4}C_{3} \times {}^{9}C_{4} + {}^{4}C_{2} \times {}^{9}C_{5} + {}^{4}C_{1} \times {}^{9}C_{6} + {}^{4}C_{0} \times {}^{9}C_{7}$$

$$= \frac{4!}{3!1!} \times \frac{9!}{4!5!} + \frac{4!}{2!2!} \times \frac{9!}{5!4!} + \frac{4!}{1!3!} \times \frac{9!}{6!3!} + \frac{4!}{0!4!} \times \frac{9!}{7!2!}$$

$= 504 + 756 + 336 + 36$

$= 1632$