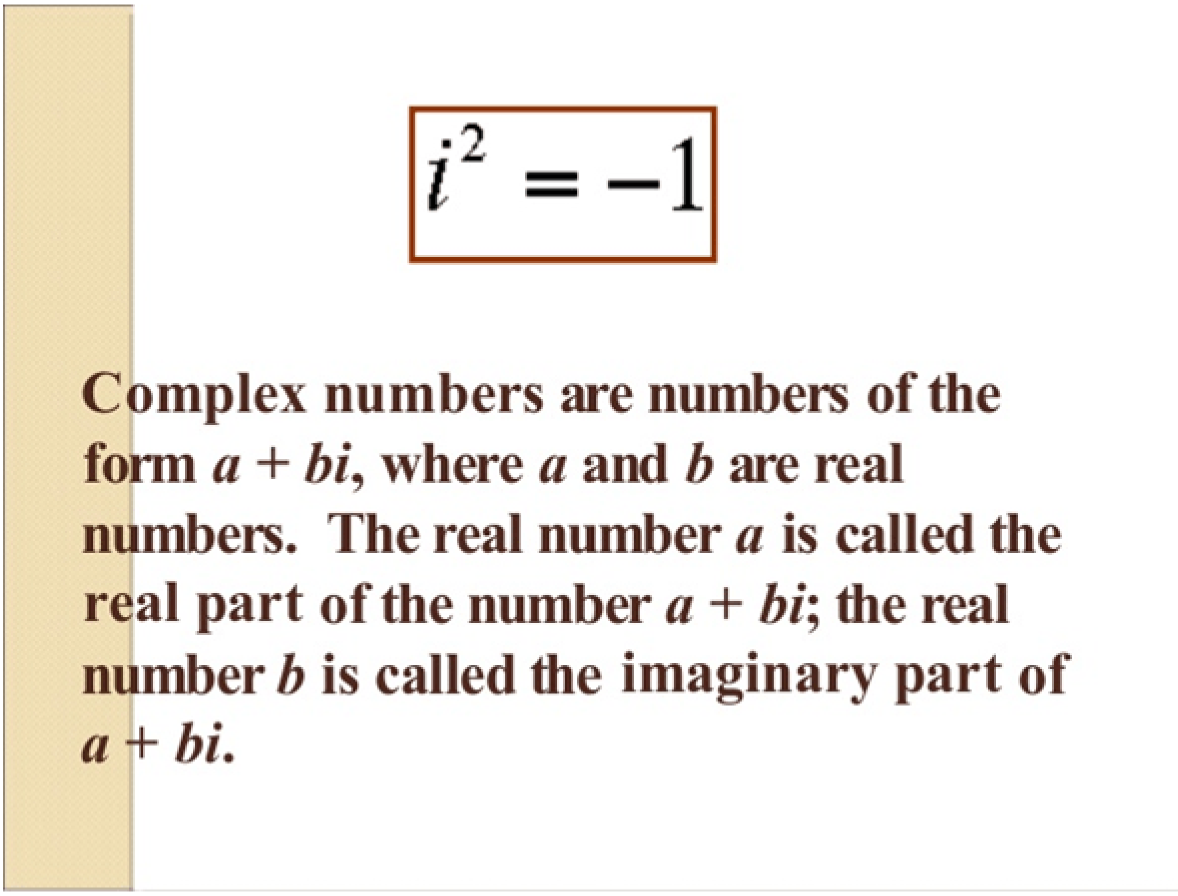
FlexiPrep

NCERT Class 11 Mathematics Solutions: Chapter 5 – Complex Number and Quadratic Equations Miscellaneous Exercise Part 2 (For CBSE, ICSE, IAS, NET, NRA 2022)



1. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

Answer:

$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)=\left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$

$$=\left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right]$$

$$=\left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$

$$=\left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right]$$

$$=\frac{33+31i}{28-10i}$$

$$=\frac{33+31i}{2(14-5i)}$$

$$=\frac{(33+31i)}{2(14-5i)}\times\frac{(14+5i)}{(14+5i)}$$ [On multiplying numerator and denominator by $(14+5i)$]

$$=\frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]}$$

$$=\frac{307+599i}{2\left(196-25i^2\right)}$$

$$=\frac{307+599i}{2(221)}=\frac{307+599i}{442}$$

$$=\frac{307}{442}+\frac{599i}{442}$$

This is the required standard form.

2. If $x-\text{iy}=\sqrt{\frac{a-\text{ib}}{c-\text{id}}}$ prove that $\left(x^2+y^2\right)^2=\frac{a^2+b^2}{c^2+d^2}$

Answer:

$$x-\text{iy}=\sqrt{\frac{a-\text{ib}}{c-\text{id}}}$$

$$=\sqrt{\frac{a-\text{ib}}{c-\text{id}}\times\frac{c+\text{id}}{c+\text{id}}}$$ [On multiplying numerator and denominator by $(c+\text{id})$]

$$=\sqrt{\frac{(\text{ac}+\text{bd})+i(\text{ad}-\text{bc})}{c^2+d^2}}$$

$$\therefore (x-\text{iy})^2=\frac{(\text{ac}+\text{bd})+i(\text{ad}-\text{bc})}{c^2+d^2}$$

$$\Rightarrow x^2 - y^2 - 2ixy = \frac{(ac + bd) + i\,(ad - bc)}{c^2 + d^2}$$

On comparing real and imaginary parts

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2}, -2xy = \frac{ad - bc}{c^2 + d^2} \text{ ... eq (1)}$$

$$\Rightarrow \left(x^2 + y^2\right)^2 = \left(x^2 - y^2\right)^2 + 4x^2y^2$$

$$\Rightarrow \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \text{ [Using (1)]}$$

$$= \frac{a^2c^2 + b^2d^2 + 2acdb + a^2d^2 + b^2c^2 - 2\,adbc}{\left(c^2 + d^2\right)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{\left(c^2 + d^2\right)^2}$$

$$= \frac{a^2\left(c^2 + d^2\right) + b^2\left(c^2 + d^2\right)}{\left(c^2 + d^2\right)^2}$$

$$= \frac{a^2 + b^2}{c^2 + d^2}$$

So, proved.

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