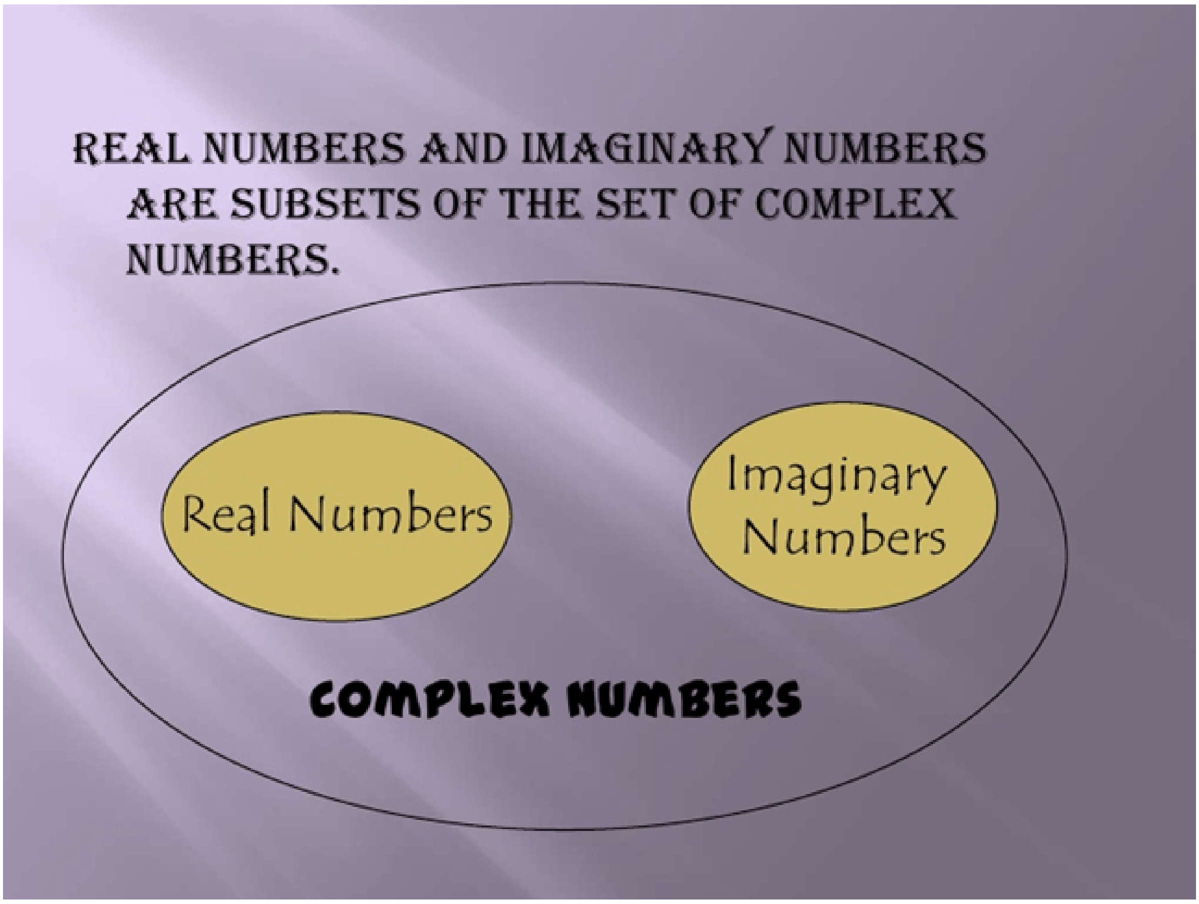
NCERT Class 11 Mathematics Solutions: Chapter 5 – Complex Number and Quadratic Equations Miscellaneous Exercise Part 1



1. Evaluate $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$

Answer:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3=\left[i^{4\times 4+2}+\frac{1}{i^{4\times 6+1}}\right]^3$$

$$=\left[\left(i^4\right)^4\bullet i^2+\frac{1}{\left(i^4\right)^6\bullet i}\right]^3$$

$$=\left[i^2+\frac{1}{i}\right]^3\ \left[\because i^4=1\right]$$

$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3\ \left[\because i^2=-1\right]$$

$$=\left[-1+\frac{i}{i^2}\right]^3$$

$$=[-1-i]^3$$

$$=(-1)^3[1+i]^3$$

$$=-\left[1^3+i^3+3\bullet 1\bullet i\,(1+i)\right]$$

$$=-\left[1+i^3+3i+3i^2\right]$$

$$=-[1-i+3i-3]$$

$$=-[-2+2i]$$

$$=2-2i$$

2. For any two complex numbers $z_1 \text{and} z_2$, prove that $\operatorname{Re}(z_1 z_2)=\operatorname{Re} z_1 \operatorname{Re} z_2-\operatorname{Im} z_1 \operatorname{Im} z_2$

Answer:

Let $z_1=x_1+iy_1 \text{and} z_2=x_2+iy_2$

$$\therefore z_1 z_2=(x_1+iy_1)(x_2+iy_2)$$

$$=x_1(x_2+iy_2)+iy_1(x_2+iy_2)$$

$$=x_1x_2+ix_1y_2+iy_1x_2+i^2y_1y_2$$

$$=x_1x_2+ix_1y_2+iy_1x_2-y_1y_2\ \left[i^2=-1\right]$$

$$=(x_1x_2-y_1y_2)+i(x_1y_2+y_1x_2)$$

$$\rightarrow \operatorname{Re}(z_1z_2)=x_1x_2-y_1y_2$$

$$\rightarrow \operatorname{Re}(z_1z_2)=\operatorname{Re} z_1 \operatorname{Re} z_2-\operatorname{Im} z_1 \operatorname{Im} z_2$$

So, proved.