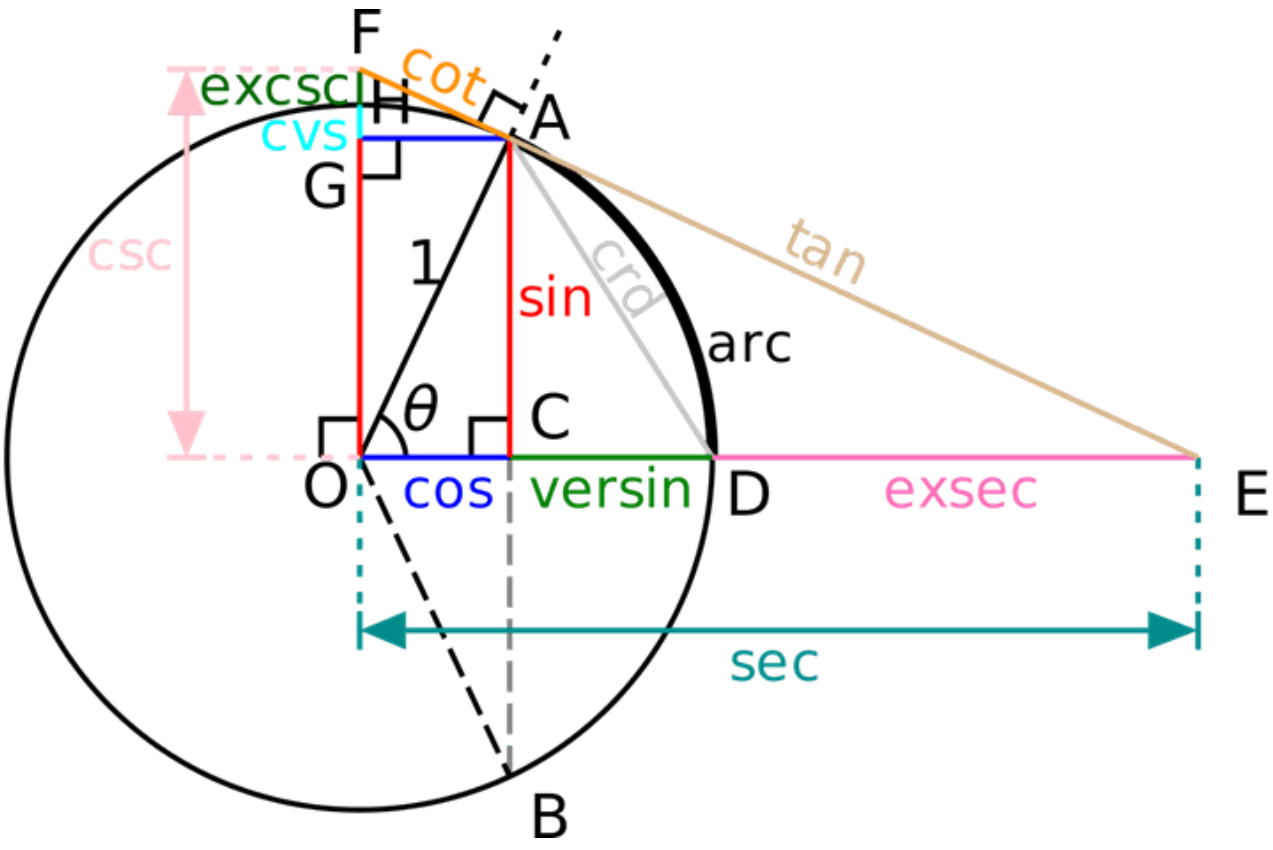
NCERT Class 11 Mathematics Solutions: Chapter 3 – Trigonometric Functions Miscellaneous Exercise Part 1



1. Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Answer:

L. H. S.

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)\left[\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{-\pi}{13}$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$=2\cos\frac{\pi}{13}\times2\times0\times\cos\frac{5\pi}{26}$$

$$=0=R.H.S$$

2. Prove that: $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x=0$

Answer:

L. H. S

$$=(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x$$

$$=\sin 3x\sin x+\sin^2 x+\cos 3x\cos x-\cos^2 x$$

$$=\cos 3x\cos x+\sin 3x\sin x-\left(\cos^2 x-\sin^2 x\right)$$

$$=\cos(3x-x)-\cos 2x\,[\cos(A-B)=\cos A\cos B+\sin A\sin B]$$

$$=\cos 2x-\cos 2x$$

$$=0$$

$$=R.H.S$$

3. Prove that: $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Answer:

L. H. S.

$$(\cos x+\cos y)^2+(\sin x-\sin y)^2$$

$$=\cos^2 x+\cos^2 y+2\cos x\cos y+\sin^2 x+\sin^2 y-2\sin x\sin y$$

$$=\left(\cos^2 x+\sin^2 x\right)+\left(\cos^2 y+\sin^2 y\right)+2(\cos x\cos y-\sin x\sin y)$$

$= 1 + 1 + 2\cos(x + y)\,[\cos(A + B) = (\cos A \cos B - \sin A \sin B)]$

$= 2 + 2\cos(x + y)$

$= 2\,[1 + \cos(x + y)]$

$= 2\left[1 + 2\cos^2\left(\frac{x + y}{2}\right) - 1\right]\,[\cos 2A = 2\cos^2 A - 1]\cos 2A$

$= 4\cos^2\left(\frac{x + y}{2}\right) - 1$

$= R.H.S$

3. Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\frac{x - y}{2}$

Answer:

L. H. S.

$= (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 y - 2\sin x \sin y$

$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) - 2\,[\cos x \cos y + \sin x \sin y]$

$= 1 + 1 - 2\,[\cos(x - y)]\,[\cos(A - B) = \cos A \cos B + \sin A \sin B]$

$= 2\,[1 - \cos(x - y)]$

$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x - y}{2}\right)\right\}\right]\,[\cos 2A = 1 - 2\sin^2 A]$

$= 4\sin^2\left(\frac{x - y}{2}\right) = R.H.S$