NCERT Class 11 Mathematics Solutions: Chapter 13 – Limits and Derivatives Miscellaneous Exercise Part 9

Basic Differentiation Rules for Elementary Functions

1. $\frac{d}{dx}[cu] = cu'$	2. $\frac{d}{dx}[u \pm v] = u' \pm v'$	3. $\frac{d}{dx}[uv] =$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$	5. $\frac{d}{dx}[c] = 0$	6. $\frac{d}{dx}[u^n] =$
7. $\frac{d}{dx}[x] = 1$	8. $\frac{d}{dx}[\lvert u \rvert] = \frac{u}{\lvert u \rvert}(u'), \quad u \neq 0$	9. $\frac{d}{dx}[\ln u]$
10. $\frac{d}{dx}[e^u] = e^u u'$	11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$	12. $\frac{d}{dx}[a^u] =$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$	14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$	15. $\frac{d}{dx}[\tan u]$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$	17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$	18. $\frac{d}{dx}[\csc u$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$	20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$	21. $\frac{d}{dx}[\text{arctan}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1+u^2}$	23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{\lvert u \rvert \sqrt{u^2-1}}$	24. $\frac{d}{dx}[\text{arccs}$

1. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers) : $\frac{\sec x - 1}{\sec x + 1}$

Answer:

$$f(x)=\frac{\sec x-1}{\sec x+1}$$

$$f(x)=\frac{\frac{1}{\cos x}-1}{\frac{1}{\cos x}+1}$$

$$=\frac{1-\cos x}{1+\cos x}$$

By quotient rule,

$$f'(x)=\frac{(1+\cos x)\frac{d}{dx}(\sin x)-(1-\cos x)\frac{d}{dx}(-\sin x)}{(1+\cos x)^2}$$

$$=\frac{\sin x+\cos x\sin x+\sin x-\sin x\cos x}{(1+\cos x)^2}$$

$$=\frac{2\sin x}{(1+\cos x)^2}$$

$$=\frac{2\sin x}{\left(1+\frac{1}{\sec x}\right)^2}$$

$$=\frac{2\sin x}{\frac{(\sec x+1)^2}{\sec^2 x}}$$

$$=\frac{2\sin x\sec^2 x}{(\sec x+1)^2}$$

$$=\frac{\frac{2\sin x}{\cos x}\sec x}{(\sec x+1)^2}$$

$$=\frac{2\sec x\tan x}{(\sec x+1)^2}$$

2. Find the derivative of the following functions (it is to be understood that $a, b, c, d, p, q, r \text{and} s$ are fixed non-zero constants and $m \text{and} n$ are integers) : $\sin^n x$

Answer:

$$y=\sin^n x$$

Accordingly, $n=1, y=\sin x$

$$\therefore \frac{dy}{dx}=\cos x, i.e., \frac{d}{dx}\sin x=\cos x$$

$$n=2, y=\sin^2 x$$

$$\therefore \frac{dy}{dx}=\frac{d}{dx}\left(\sin x\sin x\right)$$

$=(\sin x)'\sin x+\sin x(\sin x)'$ [By Leibnitz product rule]

$$=\cos x\sin x+\sin x\cos x$$

$=2\sin x\cos x$ ... eq (1)

$$n=3, y=\sin^3 x$$

$$\therefore \frac{dy}{dx}=\frac{d}{dx}(\sin x\sin^2 x)$$

$=(\sin x)'\sin^2 x+\sin x\left(\sin^2 x\right)'$ [By Leibnitz product rule]

$=\cos x\sin^2 x+\sin x(2\sin x\cos x)$ [Using (1)]

$$=\cos x\sin^2 x+2\sin^2 x\cos x$$

$$=3\sin^2 x\cos x$$

$$\frac{d}{dx}(\sin^n x)=nsin^{(n-1)}x\cos x$$

Consider $n=k$

$\frac{d}{dx}\left(\sin^k x\right)=k\sin^{(k-1)}x\cos x$ ... eq (2)

Consider,

$$\frac{d}{dx}\left(\sin^{k+1} x\right)=\frac{d}{dx}\left(\sin x\sin^k x\right)$$

$$=(\sin x)'\sin^k x+\sin x\left(\sin^k x\right)'$$

$= (\sin x)' \sin^k x + \sin x \left(k \sin^{(k-1)} x \cos x\right.$

$= \cos x \sin^k x + k \sin^k x \cos x$

$= (k+1) \sin^k x \cos x$

So, our assertion is true for $n = k + 1$.

So, by mathematical induction, $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin^n x) = n \sin^{(n-1)} x \cos x$