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NCERT Class 11 Mathematics Solutions: Chapter 13 – Limits and Derivatives Miscellaneous Exercise Part 9

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Basic Differentiation Rules for Elementary Functions

$$1. \frac{d}{dx}[cu] = cu'$$

$$2. \frac{d}{dx}[u \pm v] = u' \pm v'$$

$$3. \frac{d}{dx}[uv] =$$

$$4. \frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

$$5. \frac{d}{dx}[c] = 0$$

$$6. \frac{d}{dx}[u^n] =$$

$$7. \frac{d}{dx}[x] = 1$$

$$8. \frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$$

$$9. \frac{d}{dx}[\ln u] =$$

$$10. \frac{d}{dx}[e^u] = e^u u'$$

$$11. \frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$$

$$12. \frac{d}{dx}[a^u] =$$

$$13. \frac{d}{dx}[\sin u] = (\cos u)u'$$

$$14. \frac{d}{dx}[\cos u] = -(\sin u)u'$$

$$15. \frac{d}{dx}[\tan u] =$$

$$16. \frac{d}{dx}[\cot u] = -(\csc^2 u)u'$$

$$17. \frac{d}{dx}[\sec u] = (\sec u \tan u)u'$$

$$18. \frac{d}{dx}[\csc u] =$$

$$19. \frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$20. \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$21. \frac{d}{dx}[\arctan u] =$$

$$22. \frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$23. \frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$24. \frac{d}{dx}[\operatorname{arccsc} u] =$$

1. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and n are fixed non-zero constants and m and n are integers): $\frac{\sec x - 1}{\sec x + 1}$

Answer:

$$f(x) = \frac{\sec x - 1}{\sec x + 1}$$

$$f(x) = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1}$$

$$= \frac{1 - \cos x}{1 + \cos x}$$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(1 + \cos x) \frac{d}{dx}(\sin x) - (1 - \cos x) \frac{d}{dx}(-\sin x)}{(1 + \cos x)^2} \\ &= \frac{\sin x + \cos x \sin x + \sin x - \sin x \cos x}{(1 + \cos x)^2} \\ &= \frac{2 \sin x}{(1 + \cos x)^2} \\ &= \frac{2 \sin x}{\left(1 + \frac{1}{\sec x}\right)^2} \\ &= \frac{2 \sin x}{\frac{(\sec x + 1)^2}{\sec^2 x}} \\ &= \frac{2 \sin x \sec^2 x}{(\sec x + 1)^2} \\ &= \frac{\frac{2 \sin x}{\cos x} \sec x}{(\sec x + 1)^2} \\ &= \frac{2 \sec x \tan x}{(\sec x + 1)^2} \end{aligned}$$

2. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and n are fixed non-zero constants and m and n are integers): $\sin^n x$

Answer:

$$y = \sin^n x$$

Accordingly, $n = 1, y = \sin x$

$$\therefore \frac{dy}{dx} = \cos x, \text{ i.e., } \frac{d}{dx} \sin x = \cos x$$

$$n = 2, y = \sin^2 x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} (\sin x \sin x)$$

$$= (\sin x)' \sin x + \sin x (\sin x)' \quad [\text{By Leibnitz product rule}]$$

$$= \cos x \sin x + \sin x \cos x$$

$$= 2 \sin x \cos x \quad \dots \text{eq (1)}$$

$$n = 3, y = \sin^3 x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} (\sin x \sin^2 x)$$

$$= (\sin x)' \sin^2 x + \sin x (\sin^2 x)' \quad [\text{By Leibnitz product rule}]$$

$$= \cos x \sin^2 x + \sin x (2 \sin x \cos x) \quad [\text{Using (1)}]$$

$$= \cos x \sin^2 x + 2 \sin^2 x \cos x$$

$$= 3 \sin^2 x \cos x$$

$$\frac{d}{dx} (\sin^n x) = n \sin^{(n-1)} x \cos x$$

Consider $n = k$

$$\frac{d}{dx} (\sin^k x) = k \sin^{(k-1)} x \cos x \quad \dots \text{eq (2)}$$

Consider,

$$\frac{d}{dx} (\sin^{k+1} x) = \frac{d}{dx} (\sin x \sin^k x)$$

$$= (\sin x)' \sin^k x + \sin x (\sin^k x)'$$

$$= (\sin x)^k \sin^k x + \sin x (k \sin^{(k-1)} x \cos x$$

$$= \cos x \sin^k x + k \sin^k x \cos x$$

$$= (k + 1) \sin^k x \cos x$$

So, our assertion is true for $n = k + 1$.

So, by mathematical induction, $\frac{d}{dx}(\sin^n x) = n \sin^{(n-1)} x \cos x$