

FlexiPrep

NCERT Class 11 Mathematics Solutions: Chapter 13 – Limits and Derivatives Miscellaneous Exercise Part 12 (For CBSE, ICSE, IAS, NET, NRA 2022)

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The Quotient Rule

What about the derivative of a quotient?

Let u and v be differentiable functions and let $Q = \frac{u}{v}$. Then

$$u = Qv$$

If Q is differentiable, we have

$$\begin{aligned} u' &= (Qv)' = Q'v + Qv' \\ \implies Q' &= \frac{u' - Qv'}{v} = \frac{u'}{v} - \frac{u}{v} \cdot \frac{v'}{v} \\ \implies Q' &= \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \end{aligned}$$

This is called the **Quotient Rule**.

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1. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers) :
 $(x + \cos x)(x - \tan x)$

Answer:

$$f(x) = (x + \cos x)(x - \tan x)$$

By product rule,

$$\begin{aligned}
 f'(x) &= (x + \cos x) \frac{d}{dx}(x - \tan x) + (x - \tan x) \frac{d}{dx}(x + \cos x) \\
 &= (x + \cos x) \left[\frac{d}{dx}(x) - \frac{d}{dx}(\tan x) \right] + (x - \tan x) (1 - \sin x) \\
 &= (x + \cos x) \left[1 - \frac{d}{dx} \tan x \right] + (x - \tan x) (1 - \sin x) \\
 &= (x + \cos x) \left[1 - \frac{d}{dx} \tan x \right] + (x - \tan x) (1 - \sin x) \dots \text{eq (1)}
 \end{aligned}$$

$$g(x) = \tan x$$

Accordingly, $g(x + h) = \tan(x + h)$

By first principle,

$$\begin{aligned}
 g'(x) &= \lim_{h \rightarrow 0} \frac{g(x + h) - g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \left(\frac{\tan(x + h) - \tan x}{h} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin(x + h)}{\cos(x + h)} - \frac{\sin x}{\cos x} \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin(x + h) \cos x - \sin x \cos(x + h)}{\cos(x + h) \cos x} \right] \\
 &= \frac{1}{\cos x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin(x + h - x)}{\cos(x + h)} \right] \\
 &= \frac{1}{\cos x} \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\sin h}{\cos(x + h)} \right] \\
 &= \frac{1}{\cos x} \left(\lim_{h \rightarrow 0} \frac{\sin h}{h} \right) \left(\lim_{h \rightarrow 0} \frac{1}{\cos(x + h)} \right) \\
 &= \frac{1}{\cos x} \cdot 1 \cdot \frac{1}{\cos(x + 0)} \\
 &= \frac{1}{\cos^2 x} \\
 &= \sec^2 x \dots \text{eq (2)}
 \end{aligned}$$

So, from equation from (1) and (2)

$$\begin{aligned}
 f'(x) &= (x + \cos x) (1 - \sec^2 x) + (x - \tan x) (1 - \sin x) \\
 &= (x + \cos x) (-\tan^2 x) + (x - \tan x) (1 - \sin x)
 \end{aligned}$$

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