FlexiPrep

NCERT Class 11 Mathematics Solutions: Chapter 13 – Limits and Derivatives Miscellaneous Exercise Part 12 (For CBSE, ICSE, IAS, NET, NRA 2022)

The Quotient Rule

What about the derivative of a quotient?

Let u and v be differentiable functions and let $Q = \frac{u}{v}$. Then

$$u = Qv$$

If Q is differentiable, we have

$$u' = (Qv)' = Q'v + Qv'$$

$$\implies Q' = \frac{u' - Qv'}{v} = \frac{u'}{v} - \frac{u}{v} \cdot \frac{v'}{v}$$

$$\implies Q' = \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

This is called the **Quotient Rule**.

1. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers) : $(x + \cos x)(x - \tan x)$

Answer:

$$f(x) = (x + \cos x)(x - \tan x)$$

By product rule,

$$f'(x) = (x + \cos x)\frac{d}{dx}(x - \tan x) + (x - \tan x)\frac{d}{dx}(x + \cos x)$$

$$= (x + \cos x)\left[\frac{d}{dx}(x) - \frac{d}{dx}\left(\tan x\right)\right] + \left(x - \tan x\right)\left(1 - \sin x\right)$$

$$= (x + \cos x)\left[1 - \frac{d}{dx}\tan x\right] + (x - \tan x)(1 - \sin x)$$

$$= (x + \cos x)\left[1 - \frac{d}{dx}\tan x\right] + (x - \tan x)(1 - \sin x) \ldots \text{eq (1)}$$

$$g(x) = \tan x$$

Accordingly, $g(x + h) = \tan(x + b)$

By first principle,

$$g'(x) = \lim_{h \to 0}\frac{g(x + h) - g(x)}{h}$$

$$= \lim_{h \to 0}\left(\frac{\tan(x + h) - \tan x}{h}\right)$$

$$= \lim_{h \to 0}\frac{1}{h}\left[\frac{\sin(x + h)}{\cos(x + h)} - \frac{\sin x}{\cos x}\right]$$

$$= \lim_{h \to 0}\frac{1}{h}\left[\frac{\sin(x + h)\cos x - \sin x\cos(x + h)}{\cos(x + h)\cos x}\right]$$

$$= \frac{1}{\cos x}\lim_{h \to 0}\frac{1}{h}\left[\frac{\sin(x + h - x)}{\cos(x + h)}\right]$$

$$= \frac{1}{\cos x}\lim_{h \to 0}\frac{1}{h}\left[\frac{\sin h}{\cos(x + h)}\right]$$

$$= \frac{1}{\cos x}\left(\lim_{h \to 0}\frac{\sin h}{h}\right)\left(\lim_{h \to 0}\frac{1}{\cos(x + h)}\right)$$

$$= \frac{1}{\cos x}1 \cdot \frac{1}{\cos(x + 0)}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x \ldots \text{eq (2)}$$

So, from equation from (1) and (2)

$$f'(x) = (x + \cos x)\left(1 - \sec^2 x\right) + (x - \tan x)(1 - \sin x)$$

$$= (x + \cos x)\left(-\tan^2 x\right) + (x - \tan x)(1 - \sin x)$$