NCERT Class 11 Mathematics Solutions: Chapter 13 – Limits and Derivatives Miscellaneous Exercise Part 10

Basic Differentiation Rules for Elementary Functions

1. $\frac{d}{dx}[cu] = cu'$	2. $\frac{d}{dx}[u \pm v] = u' \pm v'$	3. $\frac{d}{dx}[uv] =$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$	5. $\frac{d}{dx}[c] = 0$	6. $\frac{d}{dx}[u^n] =$
7. $\frac{d}{dx}[x] = 1$	8. $\frac{d}{dx}[\lvert u\rvert] = \frac{u}{\lvert u\rvert}(u'), \quad u \neq 0$	9. $\frac{d}{dx}[\ln u]$
10. $\frac{d}{dx}[e^u] = e^u u'$	11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$	12. $\frac{d}{dx}[a^u] =$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$	14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$	15. $\frac{d}{dx}[\tan u$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$	17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$	18. $\frac{d}{dx}[\csc u$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$	20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$	21. $\frac{d}{dx}[\text{arctan}$
22. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$	23. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	24. $\frac{d}{dx}[\text{arccs}$

1. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers) : $\frac{a + b\sin x}{c + d\cos x}$

Answer:

$$f(x) = \frac{a + b\sin x}{c + d\cos x}$$

By quotient rule,

$$f'(x) = \frac{(c + d\cos x)\frac{d}{dx}(a + b\sin x) - (a + b\sin x)\frac{d}{dx}(c + d\cos x)}{(c + d\cos x)^2}$$

$$= \frac{(c + d\cos x)(b\cos x) - (a + b\sin x)(-d\sin x)}{(c + d\cos x)^2}$$

$$= \frac{cb\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c + d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd\left(\cos^2 x + \sin^2 x\right)}{(c + d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd}{(c + d\cos x)^2}$$

2. Find the derivative of the following functions (it is to be understood that $a, b, c, d, p, q, r, \text{and} s$ are fixed non-zero constants and $m \text{and} n$ are integers) : $\frac{\sin(x + a)}{\cos x}$

Answer:

$$f(x) = \frac{\sin(x + a)}{\cos x}$$

By quotient rule,

$$f'(x) = \frac{\cos x \frac{d}{dx}[\sin(x + a)] - \sin(x + a)\frac{d}{dx}\cos x}{\cos^2 x}$$

$$f'(x) = \frac{\cos x \frac{d}{dx}[\sin(x + a)] - \sin(x + a)(-\sin x)}{\cos^2 x} \quad \text{... eq (1)}$$

$$g(x) = \sin(x + a)$$

Accordingly,

$$g(x + h) = \sin(x + h + a)$$

By principle,

$$g'(x) = \lim_{h \to 0} \frac{g(x + h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}[\sin(x + h + a) - \sin(x + a)]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[2\cos\left(\frac{x + h + a + x + a}{2}\right)\sin\left(\frac{x + h + a - x - a}{2}\right)\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[2\cos\left(\frac{2x + 2a + h}{2}\right)\sin\left(\frac{h}{2}\right)\right]$$

$$= \lim_{h \to 0}\left[\cos\left(\frac{2x + 2a + h}{2}\right)\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\}\right]$$

$$= \lim_{h \to 0}\cos\left(\frac{2x + 2a + h}{2}\right) \cdot \lim_{h \to 0}\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\} \quad \left[\text{As } h \to 0 \Rightarrow \frac{h}{2} \to 0\right]$$

$$= \left(\cos\frac{2x + 2a}{2}\right) \times 1 \quad \left[\lim_{h \to 0}\frac{\sin h}{h} = 1\right]$$

$$= \cos(x + a) \quad \text{... eq (2)}$$

From eq (1) and (2) ,

$$f'(x) = \frac{\cos x\cos(x + a) + \sin x\sin(x + a)}{\cos^2 x}$$

$$= \frac{\cos(x + a - x)}{\cos^2 x}$$

$$= \frac{\cos a}{\cos^2 x}$$