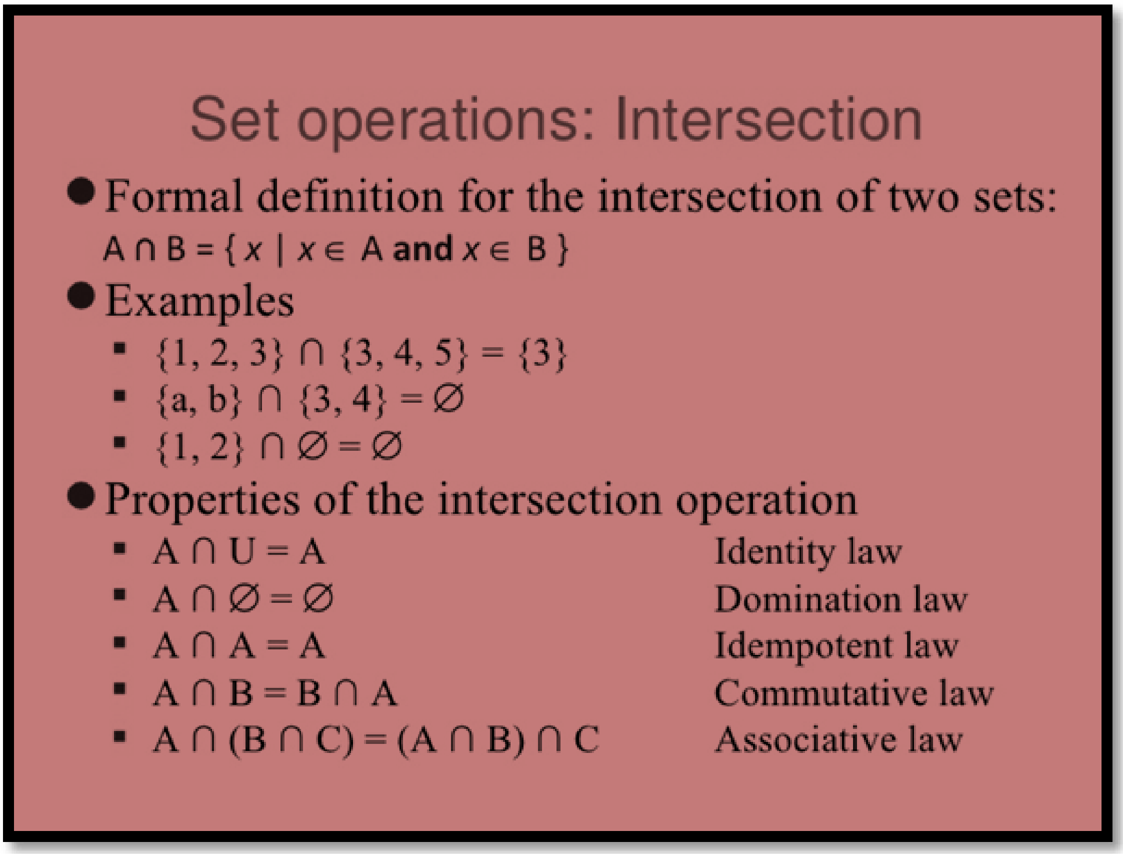
NCERT Class 11 Mathematics Solutions: Chapter 1 – Sets Miscellaneous Exercise Part 5



1. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Answer:

Consider, $A = \{0, 1\}$, $B = \{0, 2, 3\}$ and $C = \{0, 4, 5\}$

Accordingly, $A \cap B = \{0\}$ and $A \cap C = \{0\}$

Here, $A \cap B = A \cap C = \{0\}$

However, $B \neq C\,[2 \in B \text{and} 2 \notin C]$

2. Let $A \text{and} B$ be sets. If $A \cap X = B \cap X = \psi \text{and} A \cup X = B \cup X$ for some set X ,

show that $A = B$.

(Hints $A = A \cap (A \cup X)\,, B = B \cap (B \cup X)$ and use distributive law)

Answer:

Consider $A \text{and} B$ be two sets such that $A \cap X = B \cap X = f \text{and} A \cup X = B \cup X$ for

some set X to show $A = B$.

It can be seen that

$A = A \cap (A \cup X) = A \cap (B \cup X)\,[\because A \cup X = B \cup X]$

$= (A \cap B) \cup (A \cap X)$ [Distributive law]

$= (A \cap B) \cup \psi\,[\because A \cap X = \psi]$

$= A \cap B$ ... eq (1)

Now, $B = B \cap (B \cup X)$

$= B \cap (A \cup X)\,[\because A \cup X = B \cup X]$

$= (B \cap A) \cup (B \cap X)$ [Distributive law]

$= (B \cap A) \cup \psi\,[\because B \cap X = \psi]$

$= B \cap A$

$= A \cap B$ ... eq (2)

Hence, from (1) and (2) , we obtain $A = B$.

2. Find sets A, B and C such that $A \cap B, B \cap C \text{and} A \cap C$ are non-empty sets and $A \cap B \cap C = \psi$.

Answer:

Consider $A = \{0, 1\}\,, B = \{1, 2\}\,, \text{and} C = \{2, 0\}$.

Accordingly, $A \cap B = \{1\}\,, B \cap C = \{2\}\,, \text{and} A \cap C = \{0\}$.

$\therefore A \cap B, B \cap C, \text{and} A \cap C$ are non-empty.

However, $A \cap B \cap C = \psi$

3. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee.

Answer:

Consider U be the set of all students who took part in the survey.

Consider T be the set of students taking tea.

Consider c be the set of students taking coffee.

Accordingly, $n(U) = 600, n(T) = 150, n(C) = 225, n(T \cap C) = 100$

To find: Number of student taking neither tea nor coffee i.e.. ,

We have to find $n\left(T' \cap C'\right)$.

$$n\left(T' \cap C'\right) = n(T \cup C)'$$

$$= n(U) - n(T \cup C)$$

$$= n(U) - [n(T) + n(C) - n(T \cap C)]$$

$$= 600 - [150 + 225 - 100]$$

$$= 600 - 275$$

$$= 325$$

Hence, 325 students were taking neither tea nor coffee.