FlexiPrep

NCERT Class 10 Chapter 6 Pair of Linear Equations in Two Variables Official CBSE Board Sample Problems Long Answer (For CBSE, ICSE, IAS, NET, NRA 2022)

Question

4 chairs and 3 tables cost? 2100 and 5 chairs and 2 tables cost? 1750. Find the cost of one chair and one table separately.

Solution

Let the cost of one chair and one table is ′ x and y respectively.

According to question, $4r + 3y = 2100$

$5x + 2y = 1750$

Multiplying equation *(i)* by 2 and equation (ii) by 3, we get

$8x + 6y = 4200$

$15x + 6y = 5250$

Subtract ing equation (iii) from *(iv)* , we get

$$\begin{array}{r} 15x + 6y = 5250 \\ -8x - 6y = -4200 \\ \hline 7x = 1050 \Rightarrow x = 150 \end{array}$$

Putting $x = 150$ in equation *(i)* , we get

$4x150 + 3y = 2100$

$\Rightarrow 3y = 2100 - 600 \Rightarrow 3y = 1500 \Rightarrow y = 500$

Hence, cost of one chair = 150 and cost of one table = 500.

Question

Solve for x and y:

$\frac{5}{x-1} + \frac{1}{y-2} = 2$ and $\frac{6}{x-1} - \frac{3}{y-2} = 1$ [where $x \neq 1, y \neq 2$]

Solution

Given equations are

$\frac{5}{x-1} + \frac{1}{y-2} = 2 \ldots .$ (i)

$\frac{6}{x-1} - \frac{3}{y-2} = 1 \ldots . (ii)$

Let $\frac{1}{x-1} = a$ and $\frac{1}{y-2} = b$ then above system becomes.

$5a + b = 2$ ... (iii)

$6a—3b = 1$ … . (iv)

On multiplying equation *(Eli)* by 3 and then adding with equation *(ii ′)* , we get

$$15a + 3b = 6$$

$$6a - 3b = 1$$

$$21a = 7 \Rightarrow a = \frac{1}{3}$$

Putting $a = \frac{1}{3}$ in equation *(iii)* , we get

$$\frac{5}{3} + b = 2 \Rightarrow b = 2 - \frac{5}{3} \Rightarrow b = \frac{1}{3}$$

Thus, $a = \frac{1}{3}$ and $b = \frac{1}{3} \Rightarrow \frac{1}{x-1} = \frac{1}{3}$ and $\frac{1}{y-2} = \frac{1}{3}$

$$\Rightarrow x - 1 = 3 \text{ and } y - 2 = 3$$

$\Rightarrow x - 1 = 3$ and $y - 2 = 3$

$\Rightarrow x = 4$ and $y = 5$

Hence, solution of system is $x = 4$ and $y = 5$.

Question

Solve for x and y:

$$6\,(\text{ax} + \text{by}) = 3a + 2b$$

$$6\,(\text{bx} - \text{ay}) = 3b - 2a$$

Solution

Given equations arc

$6\text{ax} + 6\text{by} = 3a + 2b$ ... (i)

$6\text{bx} - 6\text{ay} = -2a + 3b$ ... (ii)

Multiplying equation *(1)* by 'a' and equation (ii) by *'b'* and then adding, we get

$6a^2 x + 6\text{aby} = 3a^2 + 2\,\text{ab}$ ... *(i)*

$6b^2 x - 6\text{aby} = -2\,\text{ab}\ \ 3b^2$ ... *(ii)*

$$6\left(a^2 + b^2\right) x = 3\left(a^2 + b^2\right)$$

Putting $x = \frac{1}{2}$ in equation *(I)* , we get

$$6a \times \frac{1}{2} + 6\text{by} = 3a + 2b \Rightarrow 2b \Rightarrow y = \frac{1}{3}.$$

Hence, solution of the system is $x = \frac{1}{2}$ and $y = \frac{1}{3}$

Question

Solve the following pair of equations by reducing them to a pair of linear equations: $\frac{1}{x} + \frac{3}{y} = 9$

$$\frac{1}{x} - \frac{4}{y} = 2$$

Solution

Given system is

$\frac{1}{x} - \frac{4}{y} = 2 \ldots \text{(i)}$

$\frac{1}{x} - \frac{3}{y} = 9 \ldots \text{(ii)}$

Iet $\frac{1}{x} = a$ and $\frac{1}{y} = b$ then the system becomes,

$a - 4b = 2 \ldots \text{(iii)}$

$a + 3b = 9 \ldots (iv)$

On multiplying equation (iii) by 3 and equation *(iv)* by 4 and then adding, we get

$3a - 12b = 6$

$4a + 12b = 36$

$7a = 42 \Rightarrow a = 6$

Putting $a = 6$ in equation *(iii)* , we get

$6 - 4b = 24b = 4b = 1$

Thus $a = 6$ and $b = 1$

$\Rightarrow \frac{1}{x} = 6 \text{ and } \frac{1}{y} = 1$

$\Rightarrow x = \frac{1}{6} \text{and } y = 1$

Hence, solution of the system is $\left(\frac{1}{6}, 1\right)$.

Question

Determine graphically whether the following pair of linear equations $2x - 3y = 5; 3x + 4y = -1$ **has**

(i) A unique solution

(ii) Infinitely many solutions or

(iii) No solution

Solution

$2x - 3y = 5$

$3y = 2x - 5$

$y = \frac{2x - 5}{3}$

X	1	4	-3
y	-1	1	-3

Pair of Linear

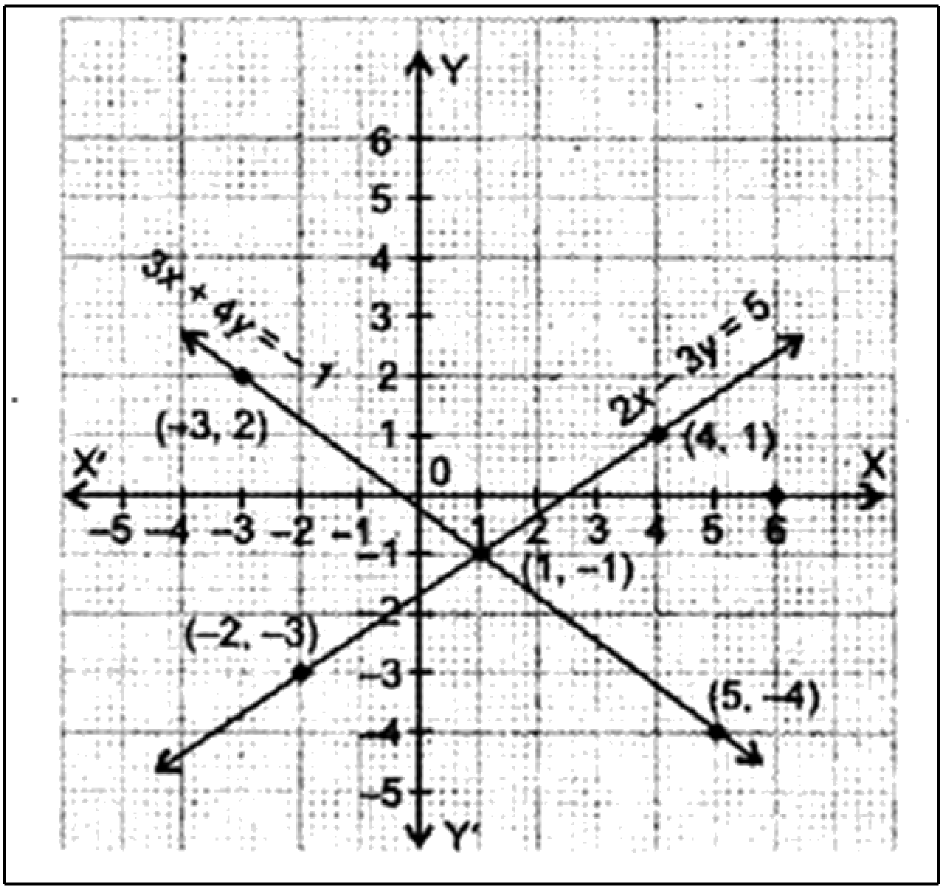
$3x - 4y = 1$

$4y = -1x - 3x$

$y = \frac{-1 - 3x}{4}$

X	1	5	-2
y	-1	-4	2

Pair of Linerar



Since the lines whose equations are given above intersect at one Point (1, -1) so, given pair of linear equations have a unique solution.

Question

In a two digit number, the digit in the unit place is twice of the digit in the tenth place. If the digits are reversed, the new number is 27 more than the given number. Find the number.

Solution

Let unit's place digit be 'x' and ten's place digit be 'y'

Then the two digit number $= 10y + x$

According to 1st condition, $x = 2y$ … (i)

On reversing the digits . f two digit number, the number becomes $10x + y$.

According to 2nd condition,

$$(10x + y) = (10y + x) + 27$$

$$10x + y - x - 10y = 27$$

$$= 9x - 9y = 27$$

$x - y = 3$ … *(ii)*

Substituting the value of x from equation *(i)* in equation (IA) , we get

$$2y - y = 3 \Rightarrow y = 3$$

Putting $y = 3$ in equation (I) , we get

$$x = 6$$

Hence, the number is 36.

Question

Solve the following pair of linear equations in two variables:

$$\frac{x}{y} + \frac{y}{b} = a + b \text{ and } \frac{x}{a^2} + \frac{y}{b^2} = 2$$

Solution

$$\text{bx} + \text{ay} = a^2 b + \text{ab}^2$$

$$b^2 x + a^2 y = 2a^2 b^2$$

On solving we get y = b2 and x = a2

Question

A boat covers 32 km upstream and 36 km downstream in 7 hours. In 9 hours it can cover 40 km upstream and 48 km downstream. Find the speed of the stream and that of the boat in still water.

Solution

Let the speed of boat in still water be x km/hr and of the stream be y km/hr.

$$\frac{32}{x - y} + \frac{36}{x + y} = 7$$

$$\frac{40}{x - y} + \frac{48}{x + y} = 7$$

Let

$$32a + 36b = 7$$

$$40a + 48b = 9$$

Solving we get:

$$b = \frac{1}{12}, a = \frac{1}{8}$$

$$x - y = 8$$

$$x + y = 12$$

Hence $x = 10, y = 2$

Speed of boat in still water = 10 km/hr

Speed of stream = 2 km/hr

Question

A man travels 600 km partly by train and partly by car. It takes 8 hours and 40 minutes if he travels 320 km by train and the rest by car. It would take 3 minutes more if he travels 200 km by train and the rest by car. Find the speed of the train and the car.

Solution

Let the speed of the train be x km/h and speed of the car be y km/h

$$\frac{320}{x} + \frac{280}{y} = 8\frac{40}{60} = \frac{52}{6}$$

$$\frac{200}{x} + \frac{400}{y} = 8\frac{70}{60} = \frac{55}{6}$$

These two equations can be simplified as:

$$320a + 280b = \frac{52}{6}$$

$$200a + 400b = \frac{55}{6}$$

On solving these two equations we get $x = 80$ km/hr $y = 60$ km/hr Speed of train = 80km/hr and speed of car = 60 km/hr

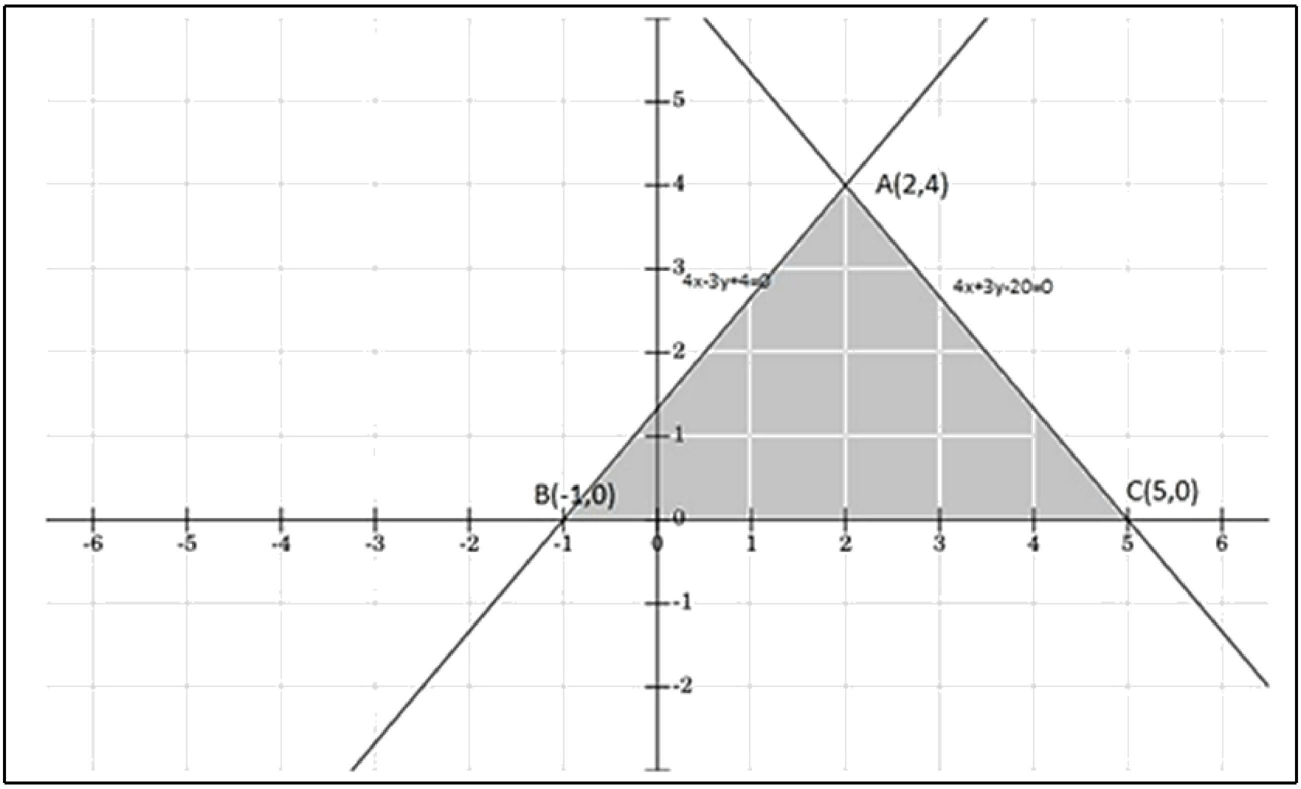
Question

Solve the following system of equations graphically:

$$4x - 3y + 4 = 0$$

$$4x + 3y - 20 = 0$$

Also find the area of the triangle formed between the lines and the x- axis.



Solution

$$x = 2, y = 4$$

Are $a = \frac{1}{2} \times 6 \times 4 = 12$ square units.

Question

Two places 'A' and 'B' are 120 km apart from each other on a highway. A car starts from 'A' and another from 'B' at the same time. If they move in the same direction, they meet in 6 hours and if they move in opposite direction, they meet in 1 hour and 12 minutes. Find the speed of each car.

Solution

Let the speed of the cars be x km/hr and y km/hr respectively where $x >$ y.

$$6x — 6y = 120x — y = 20$$

$$120$$

$$x + y = 100$$

Solving we get $x = 60$ and $y = 4O$

So the speeds of the two cars are 60km/hr and 40km/hr

Question

8 men and 12 boys can finish a piece of work in 5 days, while 6 men and 8 boys can finish it in 7 days. Find the time taken by 1 man alone and that by 1 boy alone to finish the work.

Solution

Let the time taken by a man be x days time taken by a boy be y day's work done by 1 man in one day $= \frac{1}{x}$ work done by 1 boy in one day $= \frac{1}{y}$

$$\frac{8}{x} + \frac{12}{y} = \frac{1}{5}$$

$$\frac{6}{x} + \frac{8}{y} = \frac{1}{7}$$

Let $u = \frac{1}{x} \& v = \frac{1}{y}$

$$8u + 12v = \frac{1}{5}$$

$$6u + 8v = \frac{1}{7}$$

$u = \frac{1}{70}$ and $v = \frac{1}{140}$

$x = 70$ and $y = 140$

One man alone can finish the work in 70 days and one boy alone can finish the work in 140 days.

Question

The sum of a two-digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number.

Solution

Let no at ones = x

Let no. at tens $= 10\,(x + 3)$

$$= 10 + 30$$

New no

Let no. at ones place $x + 3$

Let no. ta tens place 10x

$$10x + 30 + x + 10x + x + 3 = 121\ 22x = 88$$

$$x = \frac{88}{22}$$

$$x = 4$$

The original no. is 74

The new no. is 47

Question

Solve the following pair of equations:

$$\frac{7x-2y}{\text{xy}}$$

= 5 and

$$\frac{8x+7y}{\text{xy}}$$

= 15 where $x, y \neq 0$.

Solution

Given simultaneous equations:

$$\frac{7x-2y}{\text{xy}}=5$$

$$\Rightarrow \frac{7x}{\text{xy}}-\frac{2y}{\text{xy}}=5$$

$\Rightarrow -\frac{2}{x}=5$ … (1)

and

$$\frac{8x+7y}{\text{xy}}=15$$

$$\Rightarrow \frac{8x}{\text{xy}}+\frac{7y}{\text{xy}}=15$$

$\Rightarrow \frac{8}{y}+\frac{7}{x}=15$ … (2)

Let $\frac{1}{x}=a, \frac{1}{y}=b$

Then,

The equations becomes, $7b-2a=5$ (3)

$8b+7a=15$ … (4)

Now,

Multiply equation (3) by 7, and

Equation (4) by 2, we get

$49b-14a=35$ … (5)

$16b+14a=30-$ (6)

Add equations (5) and (6) , we

Get

$$65b=65$$

$$\Rightarrow b=\frac{65}{65}$$

$$\Rightarrow b=1$$

Substitute $b=1$ in equation (3) ,

We get

$$7b - 2 = 5$$

$$\Rightarrow 7b = 5 + 2$$

$$\Rightarrow b = \frac{7}{7}$$

$$\Rightarrow b = 1$$

Therefore,

$$a = 1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$$

and

$$b = 1 \Rightarrow \frac{1}{y} = 1 \Rightarrow y = 1$$