FlexiPrep

NCERT Class 10 Chapter 2 Polynomials Official CBSE Board Sample Problems Short Answer (For CBSE, ICSE, IAS, NET, NRA 2022)

Question

An NGO decided to distribute books and pencils to the students of a school running by some other NGO. For this they collected some amount from different people. The total amount collected is represented by $4x^4 + 2x^3 - 8x^2 + 3x - 7$ **. From this fund each student received an equal amount. The number of students, who received the amount, is represented by** $x - 2 + 2x^2$ **. After distribution,** $5x - 11$ **, amount is left with the NGO which they donated to school for their infrastructure. Find the amount received by each student from the NGO. What value has been depicted here?**

Solution

The total amount collated, $p(x)$—$4x^4 + 2x^3 - 8x^2 + 3x - 7$

Number of students, $g(x) = x - 2 + 2x^2 = 2x^2 + x - 2$

Let amount received by each students, $q(x)$.

Amount left after distribution, $r(x) = 5x - 11$

by using division algorithm, we have

$$p(x) = g(x).q(x) + r(x)$$

$$4x^4 + 2x^3 - 8x^2 + 3x - 7 = (2x^2 + x - 2)\, q(x) + (5x - 11)$$

$$\frac{(4x^4 + 2x^3 - 8x^2 + 3x - 7) - (5x - 11)}{2x^2 + x - 2} = q(x)$$

$$q(x) \frac{4x^4 + 2x^3 - 8x^2 - 2x + 4}{2x^2 + x - 2}$$

$$
\begin{array}{r}
2x^2 - 2 \\
\hline
2x^2x - 2\overline{)\ 4x^4 + 2x^3 - 8x^2 + 2x + 4\quad (} \\
\underline{-\ 4x^4 \pm 2x^3 \mp 4x^2 \quad +} \\
-4x^2 - 2x + 4 \\
\underline{-4x^2 - 2x + 4} \\
+\quad +\quad - \\
\hline
0 \\
\hline
\end{array}
$$

Amount received by each student, $q(x) = 2 - 2$

Value: humanity and socialism

Question

Divide the polynomial $x^4 - 11\,x^2 + 34x - 12$ by $x - 2$ and find the quotient and the remainder. Also verify the division algorithm.

Solution

$$
\begin{array}{r}
x^3+2x^2-13x+8 \\
x-2\overline{)\ x^4-17x^2+34x-12\ (} \\
\underline{-\ x^4 \quad \mp 2x^3} \\
2x^3-17x^2 \\
\underline{-\ 2x^3+4x^2} \\
-13x^2+34x \\
\underline{+\ -13x^2+34x} \\
8x-12 \\
\underline{-\ 8x+16} \\
4
\end{array}
$$

Let $p(x) = x^4 - 17x^2 + 34x - 12$ and $g(x) = x - 2$

Now, quotient $q(x) = x^3 + 2x62 - 13x + 8$

Remainder $r(x) = 4$

By division algoritithm

$p(x) = g(x)\,q(x) + r(x)$

$x^4 - 17x^2 + 34x - 12 = (x-2)\left(x^3 + 2x^2 - 13x + 8\right) + 4$

$= x^4 - 2x^3 - 13x^2 + 8x - 2x^2 - 4x^2 + 26x - 16 + 4$

$= x^4 - 17x^2 + 34x - 12$

Hence division algorithm is verifed.

Question

Quadratic poIynomiaI $2x^2 - 3x + 1$ has zeroes as α and . Now term a quadrille polynomial whose zeroes ase 3α and 3β .

Solution

α and β are the zeroes of the polynomial $2x^2 - 3x + 1$

$$\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{2}$$

Now, zeroes of the required polynomial arc 3α and 3β

$$\Rightarrow S = 3\alpha + 3\beta = 3(\alpha + \beta) = 3\left(\frac{3}{2}\right) = \frac{9}{2}$$

$$\Rightarrow p = (3\alpha)(3\beta) = 9(\alpha\beta) = 9 \times \frac{1}{2} = \frac{9}{2}$$

Now, required polynomial is $x^2 - 9x +$ p

$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{k}{2}\left(2x^2 - 9x + 9\right)$, Where k be any constant.

Question

On dividing $x^3 - 8x^2 + 20x - 10$ by a polynomial g (x) , the quotient and the remainder were

$x - 4$ **and 6 respectively. Find 9 (x)** .

Solution

Consider Divided: $f(x) = x^3 - 8x^2 + 20k - 10$

Quotient $q(x) = x - 4$

Remainder r *(x)* = 6 and divisor is g (x)

Applying division algorithm, we get

$$f(x) — g(x).q(x) + r(x)$$

$$\Rightarrow g(x) = \frac{f(x) - r(x)}{q(x)} = \frac{\left(x^3 - 8x^2 + 20x - 10\right) - (6)}{x - 4}$$

$$= \frac{x^3 - 8x^2 + 20x - 16}{x - 4}$$

So, $g(x) = x^2 - 4x + 4$

$$
\begin{array}{r}
x^2 - 4x + 4 \\
x - 4 \overline{)\ x^3 - 8x^2 + 20x - 16} \\
\underline{-\ x^3 \mp 4x^2} \\
-4x^2 + 20x \\
\underline{-4x^2 + 16x} \\
+\quad -\quad \\
4x - 16 \\
\underline{4x - 16} \\
-\quad + \\
0
\end{array}
$$

Question

Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verity the relationship between the zeroes and the coefficients.

Solution

Given quadratic polynomial is $3x^2 - 2$

Consider $p(x) = 3x^2 - 2$

For zeroes of polynomial p (x) , put p (x) o.

$$\Rightarrow 3x^2 - 2 = 0$$

$$\Rightarrow 3x^2 = 2$$

$$x^2 = \frac{2}{3}$$

$$\Rightarrow x = \pm\sqrt{\frac{2}{3}}$$

Hence zeroes of polynomial *p (x)* are $+\sqrt{\frac{2}{3}}$ and - $\sqrt{\frac{2}{3}}$

Here $\alpha = \sqrt{\frac{2}{3}}$ and $\beta = -\sqrt{\frac{2}{3}}$

Here sum of zeroes $= \alpha + \beta = \sqrt{\frac{2}{3}} - \sqrt{\frac{2}{3}} = 0$

Product of zeroes $= \alpha\beta = \sqrt{\frac{2}{3}} \times \left(-\sqrt{\frac{2}{3}}\right) = -\frac{2}{3}$

Also from the polynomial $p(x) = 3x^2 - 2$

Sum of zeroes $= \frac{c}{a} = \frac{0}{3} = 0$

Product of zeroes $= \frac{c}{a} = -\frac{2}{3}$

These verify the relation. _____- _____

Question

If the zeroes of a polynomial $x^2 - 8x + k = 0$, are the HCF of $(6.\ 2)$ then find the value of k.

Solution

Given, HCF of $(6,12) = 6$ is one of the roots.

$$f(6) = 0$$

$$(6)^2 - (8 \times 6) + k = 0$$

$$36 - 48 + k = 0$$

$$-12 + k = 0$$

$$k = 12$$

Question

Find the zeros of the quadratic polynomial $x^2 - 36$ and verify the relationship between the zeroes and the coefficients.

Solution

$(X^2 - 36) = (x - 6)(x + 6)$. Zeroes are 6 and -6

$$\alpha + \beta = \frac{-b}{a} = 0, \alpha\beta = \frac{c}{a} = -36$$

Question

Find the quadratic polynomial whose sum of the zeroes is 8 and one zero is $(4 + 2\sqrt{3})$

Solution

Sum = 8

Other = $8 - (4 + 2\sqrt{3})$

$$= 4 - 2\sqrt{3})$$

Product = $(4 + 2\sqrt{3})\ 8 - (4 - 2\sqrt{3})$

$$16 - 12 = 4$$

Polynomial is $x^2 - 8x + 4$

Question

If one zero of the polynomial $(a^2 + 4)\ x^2 + 13x + 4a$ is reciprocal of the other, find the value of a.

Solution

Zeroes are reciprocals of each other so the product of the zeroes is 1

$$\frac{c}{a} = 1$$

$$c = a$$

$$a^2 + 4 = 4a$$

$$a^2 — 4a + 4 = O$$

$$(a - 2)^2 = 0, a = 2$$

Question

At how many points will the polynomial $x^3 + 8$ intersect the x-axis?

Solution

It will intersect the x axis at only one point as

$$x^3 + 8 = (x + 2)\left(x^2 - 2x + 4\right)$$

$$\left(x^2 - 2x + 4\right)$$

Has no zeroes as its discriminant is less than 0