CBSE Class 10- Mathematics: Chapter – 8 Introduction to Trigonometry Part 2

Question 7:

If $\sec A = x + \frac{1}{4x}$, prove that $\sec A + \tan A = 2x \text{or} \frac{1}{2x}$.

Answer:

$$\sec \varphi = x + \frac{1}{4x}$$

$$\Rightarrow \sec^2 \varphi = \left(x + \frac{1}{4x}\right)^2 \left(\sec^2 \varphi = 1 + \tan^2 \varphi\right)$$

$$\tan^2 \varphi = \left(x + \frac{1}{4}\right)^2 - 1$$

$$\tan^2 \varphi = \left(x - \frac{1}{4}\right)^2$$

$$\tan^2 \varphi = \pm x - \frac{1}{4x}$$

$$\sec \varphi + \tan \varphi = x + \frac{1}{4x} \pm x - \frac{1}{4x}$$

$$= 2x \text{or} \frac{1}{2x}$$

Question 8:

If A, B are acute angles and $\sin A = \cos A = \cos B$, then find the value of $A + B$.

Answer:

$$A + B = 90^\circ$$

Question 9:

(a) Solve for , if $\tan 5\phi = 1$

Answer:

$$\tan 5\varphi = 1$$

$$\Rightarrow \varphi = \frac{45}{5}$$

$$\Rightarrow \varphi = 9^\circ.$$

(b) Solve for φ if $\frac{\sin\varphi}{1+\cos\varphi}+\frac{1+\cos\varphi}{\sin\varphi}=4$.

Answer:

$$\frac{\sin\varphi}{1+\cos\varphi}+\frac{1+\cos\varphi}{\sin\varphi}=4$$

$$\frac{\sin^2\varphi+1(\cos\varphi)^2}{\sin\varphi(1+\cos\varphi)}=4$$

$$\frac{\sin^2\varphi+1+\cos^2\varphi+2\cos\varphi}{\sin\varphi+\sin\varphi\cos\varphi}=4$$

$$\frac{2+2\cos\varphi}{\sin\varphi(1+\cos\phi)}=4$$

$$\Rightarrow\frac{2+(1+\cos\varphi)}{\sin\varphi(1+\cos\varphi)}=4$$

$$\Rightarrow\frac{2}{\sin\varphi}=4$$

$$\Rightarrow\sin\varphi=\frac{1}{2}$$

$$\Rightarrow\sin\varphi=\sin 30$$

$$\varphi=30^\circ$$

Question 10:

If $\frac{\cos\alpha}{\cos\beta}=m\text{and}\frac{\cos\alpha}{\sin\beta}=n,$ show that $\left(m^2+n^2\right)\cos^2\beta=n^2$

Answer:

$$\frac{\cos\alpha}{\cos\beta}=m\frac{\cos\alpha}{\sin\beta}=n$$

$$\Rightarrow m^2=\frac{\cos^2\alpha}{\cos\beta}n^2=\frac{\cos^2\alpha}{\sin^2\beta}$$

$$\text{LHS}=\left(m^2+n^2\right)\cos^2\beta$$

$$\left[\frac{\cos^2\alpha}{\cos^2\beta}+\frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$=\cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$=\frac{\cos^2\alpha}{\cos^2\beta}=n^2$$

$$\Rightarrow\left(m^2+n^2\right)\cos^2\beta=n^2$$

Question 11:

If $7\,co\sec\varphi-3\cot\varphi=7,,$ prove that $7\cot\varphi-3\text{cosec}\varphi=3$

Answer:

$$7\,co\sec\varphi-3\cot\varphi=7$$

P. T $\cot\varphi - 3\text{cosec}\varphi = 3$

$7\,co\sec\varphi - 3\cot\varphi = 7$

$\Rightarrow 7\,co\sec\varphi - 7 = 3\cot\varphi$

$\Rightarrow 7\,(co\sec\varphi - 1) = 3\cot\varphi$

$\Rightarrow 7\,(co\sec\varphi - 1)\,(co\sec\varphi + 1) = 3\cot\varphi\,(co\sec\varphi + 1)$

$\Rightarrow 7\left(\cos ec^2\varphi - 1\right) = 3\cot\varphi\,(co\sec\varphi + 1)$

$\Rightarrow 7 = 3\cot\varphi\,(co\sec\varphi + 1)$

$7\cot\varphi - 3\text{cosec}\varphi = 3$