CBSE Class 10- Mathematics: Chapter – 8 Introduction to Trigonometry Part 1

Introduction to Trigonometry

"The mathematician is fascinated with the marvelous beauty of the forms he constructs, and in their beauty he everlasting truth."

Question 1:

If $x\cos\theta - y\sin\theta = a, x\sin\theta + y\cos\theta = b,$ prove that $x^2 + y^2 = a^2 + b^2$.

Answer:

$$x\cos\theta - y\sin\theta = a$$

$$x\sin\theta + y\cos\theta = b$$

Squaring and adding

$x^2 + y^2 = a^2 + b^2$.

Question 2:

Prove that $\sec^2\theta + \cos ec^2\theta$ can never be less than 2.

Answer:

$S.T\ \sec^2\theta + \cos ec^2\theta$ can never be less than 2.

If possible, let it be less than 2.

$1 + Tan^2\theta + 1 + \cot^2\theta < 2$.

$$\Rightarrow 2 + \tan^2\theta + \cot^2\theta$$

$$\Rightarrow (\tan\theta + \cot\theta)\,2 < 2$$

Which is not possible.

Question 3:

If $\sin\varphi = \frac{1}{2},$ show that $3\cos\varphi - 4\cos^3\varphi = 0$

Answer:

$$\sin\phi = \frac{1}{2}$$

$$\Rightarrow \phi = 30^\circ$$

Substituting in place of $\phi = 30°$. We get .

Question 4:

If $\sin^2\varphi + 3\cos^2\varphi = 4S.T.$, show that $\tan\varphi = \frac{1}{\sqrt{3}}$

Answer:

If $7\sin^2\varphi + 3\cos^2\varphi = 4S.T., \tan\varphi\frac{1}{\sqrt{3}}$

$$7\sin^2\varphi + 3\cos^2\varphi = 4\left(\sin^2\varphi + \cos^2\varphi\right)$$

$$\Rightarrow 3\sin^2\phi = \cos^2\varphi$$

$$\Rightarrow \frac{\sin^2\varphi}{\cos^2\varphi} = \frac{1}{3}$$

$$\Rightarrow \tan^2\varphi = \frac{1}{3}$$

$$\tan\varphi = \frac{1}{\sqrt{3}}$$

Question 5:

If $\cos\varphi + \sin\varphi = \sqrt{2}\cos\varphi$, prove that $\cos\varphi - \sin\varphi = \sqrt{2}\sin\varphi$

Answer:

$$\cos\varphi + \sin\varphi = \sqrt{2}\cos\varphi$$

$$\Rightarrow (\cos\varphi + \sin\varphi)^2 = 2\cos^2\varphi$$

$$\Rightarrow \cos^2\varphi + \sin^2\varphi + 2\cos\varphi\sin\varphi = 2\cos^2\varphi$$

$$\Rightarrow \cos^2\varphi - 2\cos\varphi\sin\varphi + \sin^2\varphi = 2\sin^2\varphi$$

$$\Rightarrow (\cos\varphi - \sin\varphi)^2 = 2\sin^2\varphi$$

$$\left[\begin{array}{c} \therefore 2\sin^2\varphi = 2 - 2\cos^2\varphi \\ 1 - \cos^2\varphi = \sin^2\varphi \;\&\; 1 - \sin^2\varphi = \cos^2\varphi \end{array}\right]$$

$$\text{or}\cos\varphi - \sin\varphi = \sqrt{2}\sin\varphi.$$

Question 6:

If $\tan A + \sin A = m\text{and}\tan A - \sin A = n, \text{show that} m^2 - n^2 = \sqrt{mn}$

Answer:

$$\tan A + \sin A = m\tan A - \sin A = n$$

$$m^2 - n^2 = \sqrt{mn}.$$

$$= m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$

$$= 4\tan A\sin A$$

RHS $4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^4 A}{\cos^2 A}}$$

$$= 4\frac{\sin^2 A}{\cos^2 A}$$

$$= 4 \tan A \sin A$$

$$\therefore m^2 - n^2 = 4\sqrt{mn}$$