PHYSICS

PAPER - I

SECTION A

1. Answer any four of the following

(4 × 10 = 40)

(a) The Lagrangian for a system with two degrees of freedom is given by
$L = x^2 + y^2 + xy - x^2 - y^2$ (x and y being Cartesian)
Show that the system executes simple harmonic motion with respect to the generalized coordinates x and y and hence find the angular frequency of oscillation.

(b) Obtain the 2 × 2 matrix for refraction through a spherical surface.

(c) Using Fermat's principle obtain the Snell's law of refraction for light incident on a plane surface.

(d) Explain what is optical activity. How is it quantified? What are dextro rotatory and leavo rotatory crystals?
A 25 cm long tube containing sugar solution is placed between two crossed Nicols and illuminated by light of wavelength $\lambda = 6500$ Å. If the specific rotation is 40° and the optical rotation produced is 10°, what is the strength of the solution?

(e) Show that a rigid body has only six degrees of freedom. How are these six degrees of freedom chosen? What is the relevance of Euler's angles in this connection?

2. (a) Derive the following equation of motion for a rocket which suffers variation in mass during its flight:

$$\frac{Mdv}{dt} = F_{ext} + (u + v)\left(-\frac{dM}{dt}\right)$$

Here M is the mass of the rocket at any instant of time t, u is the velocity of ejected gas mass and v is the velocity of the rocket.

(20)

(b) Show that the two body central force problem can be reduced to an equivalent one body problem.

(20)

3. (a) A beam of particles of half life 2.8×10^{-8}s travels in the laboratory frame with a speed of 0.96 times the velocity of light. How much does the beam travel before the particle flux falls to 1/4 of its initial value?

(20)

(b) Show that there are two positions of a converging lens to form real images of an object on a screen kept at a fixed distance D such that D > 4f. Hence, prove that the size of the object is given by $\sqrt{h_1 h_2}$, where h_1 and h_2 are the sizes of the two real images.

(20)

4. (a) Explain unpolarised, linearly polarized and circularly polarized light. How is unpolarised light converted into circularly polarized light and analysed?

(20

(b) Derive an expression for the fineness coefficient (coefficient de finesse) in the multiple beam interferometry.

(20)

SECTION - B

5. Answer any four of the following:

(4 × 10 = 40)

(a) Consider a neutral system of point charges $Q_1, Q_2, Q_3, \ldots Q_i \ldots$ which are located at the end points of a set of vectors $\bar{r}_1, \bar{r}_2, \ldots \bar{r}_i \ldots$ drawn from the origin of a coordinate system. Show that the dipole moment $\mu = \Sigma Q_i \bar{r}_i$ is independent of the origin of the coordinate system.

(b) Two reversible heat engines E_1 and E_2 are connected in series between a hot reservoir at a temperature T_1 of 600 K and a cold reservoir at a temperature T_2 of 300 K. Engine E_1 receives 500 kJ of heat from the hot reservoir. If η_1, and η_2 are the efficiencies of E_1 and E_2, respectively, are of the same magnitude, determine

- (i) the temperature at which heat is rejected by E_1,
- (ii) work done by E_1 and E_2,
- (iii) heat rejected by engine E_2 to the cold reservoir, and
- (iv) efficiencies η_1 and η_2 of the engines.

(c) Explain why adiabatic demagnetization of a paramagnetic material causes cooling. What is the practical application of this phenomenon?

(d) The hysteresis loop of a specimen of iron weighing 10 kg is equivalent in area to 250 J/m^3 of iron. Find the loss of energy per hour at the rate of 50 Hz/s. Assume density of iron as 7.5 kg/m^3.

(e) Find the form of Maxwell's equations in terms of E and H for a linear and isotropic medium.

6. (a) The potential due to a linear quadrupole is given by

$$V = Q_d \frac{(3\cos^2\theta - 1)}{16\pi\varepsilon_0 \gamma^3},$$

where Q_d is a constant. Compute the electrostatic field for this potential.

(20)

(b) At a given instant, a certain system has a current density given by

$$\bar{J} = A\left(x^3\hat{x} + y^3\hat{y} + z^3\hat{z}\right)$$

where A is positive constant.

- (i) In what units will A be measured?
- (ii) At this instant, what is the rate of change of the charge density at the point (2, -1, 4) metre?
- (iii) Consider the total charge Q contained within a sphere of radius a centred at the origin. At this instant, what is the rate at which Q is changing in time?

(20)

7. (a) What do you understand by covariance of Maxwell's equations? Write the co-variant F_{ik} and contravariant F_{ik} comlionents of electromagnetic field tensor in matrix form.

(20)

(b) Give the proof of Stefan-Boltzman law from thermodynamic considerations.

(20)

8. (a) Discuss with necessary theory the J—K effect for a van der Waal's gas. What is the practical application of the J—K (Joule-Kelvin effect)?

(20)

(b) Derive an expression for the average energy of an oscillator on the basis of Einstein's theory. Show that at high temperatures

$$\bar{E} = kT.$$

How Debye modified Einstein's Theory?

(20)

Useful Constants

Electron charge	: $e = 1.602 \times 10^{-19}$ coul.
Electron mass	: $m_0 = 9.109 \times 10^{-31}$ kg
Vaccum permittivity	: $\varepsilon_0 = 8.854 \times 10^{-12}$ Farad/m
Vacuum permeability	: $\mu_0 = 1.257 \times 10^{-6}$ Henry/m
Velocity of light	: $c = 3 \times 10^{8}$ m/s
Electron volt	: $eV = 1.602 \times 10^{-19}$ J
Boltzmarin Constant	: $k = 1.38 \times 10^{-23}$ j/°K.

PHYSICS

PAPER - II

SECTION A

Useful Constants

Mass of proton	$= 1.673 \times 10^{-27}$ kg
Mass of neutron	$= 1.675 \times 10^{-27}$ kg
Mass of electron	$= 9.11 \times 10^{-31}$ kg
Planck constant	$= 6.626 \times 10^{-34}$ Js
Boltzmann Constant	$= 1.380 \times 10^{-23}$ Jk^{-1}
Bohr magneton	$= 9.273 \times 10^{-24}$ A/m^2
Electronic charge	$= 1.602 \times 10^{-19}$ C
Atomic mass unit (amu)	$= 1.660 \times 10^{-27}$ kg
	$= 931$ MeV

Velocity of light in vacuum, $c = 3 \times 10^8$ ms^{-1}

$m\left({}^1_1\mathrm{H}\right)$	$= 1.007825$ amu
$m\left({}^2_1\mathrm{H}\right)$	$= 2.014102$ amu
$m\left({}^3_1\mathrm{H}\right)$	$= 3.016049$ amu
$m\left({}^{12}_6\mathrm{C}\right)$	$= 12.000000$ amu
$m\left({}^{20}_{10}\mathrm{Ne}\right)$	$= 19.992439$ amu
$m\left({}^4_2\mathrm{He}\right)$	$= 4.002603$ amu

1. Answer any four of the following:

(a) The electron spin operator can be written in the matrix form in terms of the Pauli spin operator $\hat{\sigma} = 2\hat{s}$, where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show that

$$\sigma_x^2 = 1; \sigma_y^2 = 1; \sigma_z^2 = 1$$

and

$$\sigma_x\sigma_y = -\sigma_y\sigma_x = i\sigma_z$$
$$\sigma_y\sigma_z = -\sigma_z\sigma_y = i\sigma_x$$
$$\sigma_z\sigma_x = -\sigma_x\sigma_z = i\sigma_y$$

(10)

(b) State and explain the Heisenberg's uncertainty principle. Show that the natural line width of a spectral line follows from this principle.

(10)

(c) State Franck-Condon principle and discuss its applications in molecular spectroscopy.

(10)

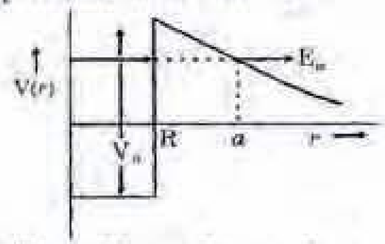
(d) Explain the role of molecular hydrogen ion spectrum in astronomy.

(10)

(e) Discuss the application of Raman effect to molecular structure.

(10)

2. (a) What is WKB approximation? The potential seen by an alpha particle inside a nucleus can be represented as shown below.



$$V(r) = -V_0 \qquad \text{for } r < R$$
$$= \frac{zZe^2}{r} \qquad \text{for } r > R$$

where R is the nuclear radius, z and Z represent the atomic numbers of the alpha particle and the nucleus. Assuming $l = 0$, obtain an expression for the lifetime for alpha decay.

(20)

(b) Write down the Schrödinger equation for a linear harmonic oscillator and obtain the energy eigenvalues.

(20)

3. (a) Describe Stern-Gerlach experiment. Explain how it demonstrates the discrete nature of the magnetic moment of an atom.

(15)

(b) What is the significance of fine structure correction? Calculate the energy shift due to spin-orbit interaction term in H-like system.

(15)

(c) Assuming LSJ coupling holds, list the possible fine structure terms for the following configurations:

(10)

(i) $2p^3$

(ii) $(1s)^2 (2s)^2 (2p)^5$

4. (a) Explain the underlying principle of nuclear magnetic resonance spectroscopy. Why does a spinning nucleus precess in a magnetic field? Describe NMR spectrometer.

(30)

(b) Show that a diatomic molecule can not have any arbitrary value of rotational energy.

(10)

SECTION B

5. Answer any four of the following

(a) Determine from the given data whether the following reactions are exothermic or endothermic :

(10)

(i) $^1_1H + ^3_1H \rightarrow ^2_1H + ^2_1H$

(ii) $^{12}_6C + ^{12}_6C \rightarrow ^{20}_{10}Ne + ^4_2He$

(b) Calculate the recoil energy of $^{57}_{26}Fe$ nucleus when it emits a gamma photon of energy 14 keV.

(10)

(c) Beam of thermal neutrons is employed in crystal structure studies. Justify this statement. (Room temperature = 300 K) (10)

(d) Which of the following reactions are permitted or not permitted by various conservation laws?

(10)

(i) $K^- + p \rightarrow \Sigma^+ + \pi^-$
(ii) $K^+ + n \rightarrow \Sigma^+ + \pi^0$
(iii) $\pi^- + p \rightarrow K^- + \Sigma^+$
(iv) $\pi^+ \rightarrow \mu^+ + \nu$

(e) On the basis of band model, explain the flow of charge carriers in a p-n diode under the influence of an external bias.

(10)

6. (a) Write down the semi-empirical mass formula for a nucleus. Explain the significance of each term occurring in it. Discuss the stability of a nucleus against β-decay. What effect of the pairing term on stability?

(25)

(b) What are mirror nuclei? How does the charge independence of nuclear forces emerge from this concept?

(15)

7. (a) Give some characteristic properties of strange particles which distinguish them from non-strange ones.

Write the Gell-Mann-Nishijima relation and show how this is used for the classification of elementary particles.

(15)

(b) What is a Joseph son junction? Explain the phenomenon of tunneling of charge carriers across such a junction. How does this tunneling differ from a single-particle tunneling between two superconductors separated by an insulator?

Discuss DC and AC Josephson effects. Derive the relation

$$\nu = \frac{2eV}{h}$$

where the symbols have their usual meanings.

(25)

8. (a) Obtain an expression for the gain of an operational amplifier in the inverting mode. Explain how it can be employed as (i) an adder, (ii) an integrator and (iii) a differentiator.

(15)

(b) Explain the functions of ALU, primary memory and secondary memory in a computer system.

(15)

(c) Find the Boolean expression for the output (X) in the following circuit :

(10)