

Obtain the following integrals as the limit of a sum:

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| <p>(1) $\int_0^1 (x^2 + 3) dx$ [Ans: $\frac{10}{3}$]</p> | <p>(2) $\int_1^3 e^x dx$ [Ans: $e^3 - e$]</p> | <p>(3) $\int_2^3 (x^2 - 2x) dx$ [Ans: $\frac{4}{3}$]</p> |
| <p>(4) $\int_1^2 3^x dx$ [Ans: $\frac{6}{\log 3}$]</p> | <p>(5) $\int_{\log 2}^{\log 5} e^x dx$ [Ans: 3]</p> | <p>(6) $\int_a^b \cos x dx$ [Ans: $\sin b - \sin a$]</p> |

Solve the following problems:

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| <p>(7) $\int_{-1}^3 2x - 1 dx$ [Ans: $\frac{17}{2}$]</p> | <p>(8) $\int_0^{2\pi} \sin mx \sin nx dx,$ $m, n \in \mathbb{N}$ [Ans: 0 if $m \neq n$, π if $m = n$]</p> | <p>(9) $\int_0^1 \sin^{-1} \sqrt{\frac{x}{x+1}} dx$ [Ans: $\frac{\pi}{2} - 1$]</p> |
| <p>(10) $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$ [Ans: $\frac{\pi}{4} - \frac{1}{2} \log 2$]</p> | <p>(11) $\int_0^{\pi} \frac{x dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ [Ans: $\frac{\pi^2}{2ab}$]</p> | <p>(12) $\int_0^{2\pi} \cos^3 x \sin^5 x dx$ [Ans: 0]</p> |
| <p>(13) P.t. $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx = \frac{\pi^2}{4}$</p> | | <p>(14) P.t. $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$</p> |
| <p>(15) P.t. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$</p> | <p>(16) P.t. $\int_0^{\pi} \frac{x dx}{1 + \sin x} = \pi$</p> | |

Solve the following problems:

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| <p>(17) If $f(x + \alpha) = f(x) \forall x \in \mathbb{R}$, i.e., if f has a period α, then prove that</p> $\int_0^{n\alpha} f(x) dx = n \int_0^{\alpha} f(x) dx, \text{ where } n \in \mathbb{N}.$ | |
| <p>(18) Prove that $\int_0^a x^2 (a - x)^n dx = \frac{2a^{n+3}}{(n+1)(n+2)(n+3)}$</p> | |
| <p>(19) If $f(n) = \int_0^{\frac{\pi}{4}} \tan^n x dx$, then prove that $f(n) + f(n-2) = \frac{1}{n-1}$, $n \neq 1$.</p> | |
| <p>(20) Evaluate: $\int_0^{\frac{\pi}{2}} \sin^4 x dx + \int_0^{\frac{\pi}{2}} \cos^4 x dx$. [Ans: $\frac{3\pi}{8}$]</p> | |
| <p>(21) $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ [Ans: $\frac{\pi}{8} \log 2$]</p> | <p>(22) $\int_1^2 \frac{x^2 + 1}{x^4 + 1} dx$ [Ans: $\frac{1}{\sqrt{2}} \tan^{-1} \frac{3}{2\sqrt{2}}$]</p> |
| <p>(23) $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$ [Ans: $\frac{\pi}{4}$]</p> | <p>(24) $\int_0^{\frac{\pi}{2}} \sin^6 x dx$ [Ans: $\frac{5\pi}{32}$]</p> |
| <p>(25) $\int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} dx$ [Ans: $\frac{\pi}{4} - \frac{1}{2}$]</p> | <p>(26) $\int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x} dx$ [Ans: $\frac{\pi}{2}$]</p> |
| <p>(27) $\int_0^{\frac{\pi}{2}} \frac{x dx}{\sin x + \cos x}$ [Ans: $\frac{\pi}{2\sqrt{2}} \log(1 + \sqrt{2})$]</p> | <p>(28) $\int_0^{\frac{\pi}{2}} \frac{dx}{(\sin x + \cos x)^2}$ [Ans: 1]</p> |

Solve the following problems:

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| (29) $\int_0^1 \frac{e^{-x}}{1+e^x} dx$ [Ans: $\log(1+e) - \frac{1}{e} - \log 2$] | (30) $\int_0^{\log 2} \sqrt{e^x - 1} dx$ [Ans: $2 - \frac{\pi}{2}$] |
| (31) $\int_1^2 \frac{dx}{(x+1)\sqrt{x^2-1}}$ [Ans: $\frac{1}{\sqrt{3}}$] | (32) $\int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ [Ans: π] |
| (33) $\int_a^b \sqrt{\frac{x-a}{b-x}} dx$ [Ans: $\frac{\pi}{2}(b-a)$] | (34) $\int_3^8 \frac{2-3x}{x\sqrt{1+x}} dx$ [Ans: $2\log\frac{3}{2} - 6$] |
| (35) $\int_1^2 \frac{\log x}{x^2} dx$ [Ans: $\frac{1}{2}(1 - \log 2)$] | (36) $\int_0^{\frac{\pi}{4}} x^2 \sin x dx$ [Ans: $-\frac{\pi^2}{16\sqrt{2}} + \frac{\pi}{2\sqrt{2}} + \sqrt{2} - 2$] |
| (37) $\int_0^{\frac{\pi}{4}} x \cos x \cos 3x dx$ [Ans: $\frac{1}{16}(\pi - 3)$] | (39) $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx, x < 1$ [Ans: $\frac{\pi}{2} - \log 2$] |
| (38) $\int_0^\pi x \sin nx dx$ [Ans: $-(-1)^n \frac{\pi}{n}$] | (41) $\int_0^\pi \frac{x dx}{1 + \cos \alpha \sin x}, 0 < \alpha < \frac{\pi}{2}$ [Ans: $\frac{\pi \alpha}{\sin \alpha}$] |
| (40) $\int_0^{\frac{1}{2}} \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$ [Ans: $-\frac{\sqrt{3}\pi}{6} + \frac{\pi}{2} - \frac{1}{2}$] | (43) $\int_8^{27} \frac{dx}{x - \sqrt[3]{x}}$ [Ans: $\frac{3}{2} \log \frac{8}{3}$] |
| (42) $\int_0^\pi \log \sin x dx$ [Ans: $-\pi \log 2$] | (45) $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ [Ans: $\frac{1}{20} \log 3$] |
| (44) $\int_0^{\frac{\pi}{4}} \sqrt{\tan x} dx$ [Ans: $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}} \log(\sqrt{2} - 1)$] | |