C.S.E.

B

MATHEMATICS–2005

(PRELIMINARY)

Time Allowed : Two Hours ***Maximum Marks : 300***

1. For what value of λ, are the the lines $x - 3y + 3, z = \lambda y - 6$ and $x = -3y + 10, z = 2y + 2$ orthogonal ?

   (a) – 5 (b) –3

   (c) 4 (d) 6

2. If a circle and a parabola (symmetrical about x- or y-axis) intersect in four points, then what is the algebraic sum of the ordinates or abscissae of the four points ?

   (a) 0 (b) 1

   (c) 3 (d) 4

3. What is the point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$?

   (a) $\vec{a} + \vec{b}$ (b) $\vec{a} - \vec{b}$

   (c) $\vec{b} - \vec{a}$ (d) $\vec{b} + \vec{a}$

4. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors, then what is the value of the determinant ?

$$\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$$

   (a) 0 (b) 1

   (c) $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ (d) $a^2 + b^2 + c^2$

5. For what value of λ, will the sum of the squares of the roots of the equation $x^2 + (2 - \lambda)x + 1 = \lambda$ have the least value?

   (a) 0 (b) 1

   (c) 2 (d) 3

6. If $f(x) = \frac{1-x}{1+x}$, $x > 0$

then what is the least value of $f\{f(x)\} + f\{f(1/x)\}$?

(a) 1 (b) 2

(c) 3 (d) 0

7. If $f(x)$ is defined over $[0, 1]$ then what is the domain of the function $f(2x + 3)$?

(a) $[-3/2, -1]$ (b) $[3, 5]$

(c) $R - \{3/2\}$ (d) $(-3/2, -1)$

8. A function f is defined such that for all real x, y

(i) $f(x + y) = f(x) \cdot f(y)$

(ii) $f(x) = 1 + x\, g(x)$

where $\lim_{x \to 0} g(x) = 1$

What is $\frac{d}{dx} f(x)$ equal to ?

(a) $g(x)$ (b) $f(x)$

(c) $g'(x)$ (d) $f(x) + xg'(x)$

9. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

where p is constant. What is the value of $\frac{d^3}{dx^3} f(x)$ at $x = 0$?

(a) 0 (b) $p + p^2$

(c) $p + p^3$ (d) $-6p^3$

10. If $f(x)$ and $g(x)$ are two functions continuous everywhere and $F(x) = \lim_{n \to \infty} \frac{f(x) + x^{2n} g(x)}{1 + x^{2n}}$ then $F(x)$ is continuous everywhere except at which of the following points ?

(a) 2 (b) −2

(c) ±1 (d) ∞

11. Let $f: R \to R$ be a monotonically decreasing function (defined everywhere on the real line) and lef f be discontinuous at certain point x = a. What is the nature of this discontinuity?

(a) Removable discontinuity

(b) Either removable discontinuity or discontinuity of 2nd kind

(c) Discontinuity of 1st kind

(d) Either discontinuity of 1st kind or of 2nd kind

12. What is the centre of the sphere having the circle $x^2 + y^2 + z^2 - 3x + 4y - 2z - 5 = 0, 5x - 2y + 4z - 7 = 0$ as the great circle ?

(a) (1, 1, 1) (b) (−1, −1, 1)

(c) (1, −1, −1) (d) (−1, 1, 1)

13. A rod of length one metre begins to slide in a vertical plane perpendicular to a wall and floor. What is the locus of a point at a distance 1/3 from one end of the rod ?

(a) A straight line of slope 45°

(b) A straight line of slope 60°

(c) A circle

(d) An ellipse

14. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and those of hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then what is the value of b^2 ?

(a) 4 (b) 5

(c) 7 (d) 9

15. Match List-I (Curve) with List-II (Nature of Origin) and select the correct answer using the codes given below the lists :

List-I	*List-II*
A. $y^2 = b(x^2 + x^3)$ if $b > 0$	1. Node

B. $y^2 = b(x^4 + x^3)$ if $b = 0$ 2. Cusp

C. $y^2 = b(x^4 + x^3)$ if $b > 0$ 3. Conjugate point

4. Neither a node nor a cusp nor a conjugate point

	A	B	C
(a)	1	4	3
(c)	1	2	3

	A	B	C
(b)	3	4	1
(d)	3	2	1

16. Resultant of a force P along x-axis, a force Q along y-axis and a couple G (taken positive in counterclockwise direction) in xy-plane acts along which one of the following lines ?

(a) $Py - Qx + G = 0$ (b) $Qy - Px - G = 0$

(c) $Qy + Px - G = 0$ (d) $Qy + Px + G = 0$

17. Equation of a common projectile motion is given to be $y = x - x^2$. What is the velocity of the projection ?

(a) $\sqrt{2g}$ (b) $\sqrt{g}$

(c) $g/2$ (d) $g/4$

18. A particle P of unit mass moves in a plane under the attractive forces from two fixed points A and B ; the magnitude of the forces being PA, PB respectively. When distance PA = PB = $\sqrt{2}$ times the perpendicular distance of the particle P from AB, the direction of velocity is parallel to AB. What is the path of the particle P ?

(a) An ellipse with A, B as foci

(b) A circle with centre at the mid-point of AB

(c) A hyperbola with A, B as foci

(d) A parabola with AB as directrix

19. If A and B are usual projectiles describing parabolic paths under gravity, then relative to A, B appears to describe :

(a) a straight line (b) a parabola

(c) an ellipse (d) a circle

20. In a simple harmonic motion, of amplitued a and period T,

what does $\int_0^T v^2 dt$ equal to (v being velocity of the motion) ?

(a) $\frac{\pi a^2}{T}$ (b) $\frac{\pi a}{2T}$

(c) $\frac{\pi a^2}{T^2}$ (d) $\frac{2\pi^2 a^2}{T}$

21. A particular is moving under the influence of a central force along the curve $r = ae^{\omega\theta}$, then force directed towards origin is proportional to :

(a) r^3 (b) r^2

(c) $1/r^3$ (d) $1/r^2$

22. If a particle moves along a straight line according to the law $s^2 = at^2 + 2bt + c$ where s is the displacement at any instant t, then how does the acceleration vary with displacement ?

(a) As $1/s$ (b) As $1/s^2$

(c) As $1/s^3$ (d) As s^3

23. For what value of f, may the forces of magnitude $f - 1$, $f + 1$ and $f + 2$, acting at a point be in equilibrium ?

(a) 1·25 (b) 1·5

(c) 1·75 (d) 2·5

24. Three forces 3, 4 and 5 units are acting at a point of a body, making equal angles with each other. What is the magnitude of their resultant ?

(a) 12 units (b) 6 units

(c) $\sqrt{3}$ units (d) 0

25. A ladder AB with the end A on a smooth vertical wall and the end B on a rough horizontal floor. If the inclination to the vertical is 45°, the friction μ between the ladder and the horizontal floor ?

(a) $\mu = \frac{1}{\sqrt{2}}$ (b) $\mu = \sqrt{2}$

(c) $\mu = 1$ (d) $\mu = \frac{1}{2}$

26. Forces of magnitude 1, 2, 3, 4, 5 N act at an angular point of a regular hexagon towards the other angular points. What is the approximate value of magnitude of their resultant ?

(a) 12 N (b) $\sqrt{55}$ N

(c) 7·5 N (d) 15 N

27. Let $m_1, m_2, \ldots, m_K$ be k mass particles at positions with position vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_K$ respectively referred to a fixed point a origin. If each moves with velocity $\vec{A} + t\vec{B}$ (where $\vec{A}$ and $\vec{B}$ are constant vectors) at time t, then which one of the following is correct ?

(a) The centre of mass of the particles moves with velocity $\vec{A}$

(b) The centre of mass of the particles moves with velocity $t\vec{B}$

(c) The centre of mass of the particles moves with velocity

$$\frac{\sum_{i=1}^{k} \vec{r}_i \times (\vec{A} + t\vec{B})}{\sum_{i=1}^{k}}$$

(d) The centre of mass of the particles moves with velocity $\vec{A} + t\vec{B}$

28. Which of the following are the generators of the multiplicative group $(\{1, 2, 3, 4, 5, 6\}, X_7)$ where X_7 denotes multiplication modulo 7 ?

(a) 3 and 4 (b) 3 and 5

(c) 4 and 5 (d) 3, 4 and 5

29. Which one of the following polynomials lies in the linear span of $S = \{1, 1 + x + x^2\}$?

(a) $5x^2 + 5x + 1$ (b) $100x^2 + 10x + 10$

(c) $\pi(x^2 - 1)$ (d) $7x^2 + \pi x + 3$

30. Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ where $ad \neq 0$, under matrix multiplication and $N = \left\{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\right\}$ where a, b, c, d are real number.

Consider the following statements :

1. N is not a subgroup of G.
2. N is a subgroup of G.
3. N is normal in G.
4. N is not normal in G.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only
(c) 2 and 3 (d) 2 and 4

31. Let R be a ring with unit element. Using its elements another ring $(\overline{R}, \oplus, \cdot)$ is formed by defining $a \oplus b = a + b + 1$ and $a \cdot b = ab + a + b$. What are the zero element and unit element of $\overline{R}$ respectively ?

(a) 1, 0 (b) 0, 1
(c) -1, 0 (d) 0, 1

32. Which one of the following is a cyclic group ?

(a) Set of non-zero square matrices of order 2×2 over R under matrix multiplication
(b) The group of roots (real or complex) of the equation $x^n - 1 = 0$ where n is a natural number
(c) The group $Q - \{0\}$ of non-zero rationals under multiplication
(d) The group Q of rationals under addition

33. G is a group of all non-zero complex numbers $a + ib$ (a, b are real but both are not zero) under multiplication and

$H = \{a + ib \mid (a^2 + b^2) = 1\}$

Consider the following statements :

1. H is a subgroup of G.
2. H is Abelian.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

34. In group theory, which one of the following statements is correct ?

(a) Abelian groups may have non-Abelian subgroups

(b) Non-Abelian groups may have Abelian subgroups

(c) Cyclic groups may have non-cyclic subgroups

(d) Non-cyclic groups cannot have cyclic subgroups

35. If G is a group of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $(ad - bc) \neq 0$ and a, b, c, d are integers modulo 3, relative to matrix multiplication, then what is the number of elements in G ?

(a) 8 (b) 16

(c) 48 (d) 81

36. For the set $\mathbb{N}$ of natural numbers, consider the following operations :

$m * n$ = l.c.m. of m and n

$m \times n$ = g.c.d. of m and n

Which one of the following is correct ?

(a) $(\mathbb{N}, *)$ is a group , but $(\mathbb{N}, \times)$ is not a group

(b) $(\mathbb{N}, \times)$ is a group, but $(\mathbb{N}, *)$ is not a group

(c) Both $(\mathbb{N}, *)$ and $(\mathbb{N}, \times)$ are groups

(d) Neither $(\mathbb{N}, *)$ nor $(\mathbb{N}, \times)$ is a group

37. What is the number of disjoint cycles of length > 1 in the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 2 & 7 & 6 & 3 & 4 & 1 \end{pmatrix}$?

(a) 2 (b) 3
(c) 4 (d) 5

38. Each root of the equation
$y^4 - 22y^3 + 130y^2 - 243y + 61 = 0$
is greater by p than each root of the equation
$x^4 - 6x^3 - 38x^2 - 3x + 17 = 0$
What is the value of p ?
(a) 16 (b) 8
(c) −4 (d) 4

39. If the roots of the equation $(x - a)(x - b) + k = 0$ are c and d, then what are the roots of the equation $(x + c)(x + d) + k = 0$?
(a) −a and −b (b) a and b
(c) −a and b (d) a and −b

40. What is the imaginary part of $\frac{(\sin 2\theta + i\cos 2\theta)^4}{(\sin\theta - i\cos\theta)^5}$?
(a) sin 11θ (b) cos 11θ
(c) −sin 11θ (d) −cos 11θ

41. What is the angle at which the curve $r^2 = 2\text{cosec } 2\theta$ intersects the line $\theta = \pi/6$?
(a) 0 (b) $\pi/3$
(c) $2\pi/3$ (d) π

42. What is the number of real roots lying in the interval (−1, 1) for the function $f(x) = 3x^5 + 15x - 8$?
(a) 1 (b) 2
(c) 3 (d) 4

43. What is the minimum value of the function
$y = \int_0^x (t - 1)(t - 2)\,dt$ where $x > 0$?
(a) 1 (b) 1/2
(c) 3/2 (d) 5/2

What is the length of an arc of the curve $r = ae^{\theta \cot\alpha}$ (taking length $s = 0$ when $\theta = 0$) ?

(a) $(r + a) \sin \alpha$ (b) $(r - a) \cos \alpha$

(c) $(r + a)$ cosec α (d) $(r - a)$ sec α

45. What is the area bounded by the curve $y = x \sin x$ and the x-axis between $x = 0$ and $x = 2\pi$?

(a) π (b) 2π

(c) 3π (d) 4π

46. What is the area of the surface of the cone generated by revolving the line segment $y = 2x$ from $x = 0$ to $x = 2$, about the x-axis ?

(a) $32\pi/3$ (b) $16\pi/3$

(c) $4\sqrt{5}\pi$ (d) $8\sqrt{5}\pi$

47. Let a be a positive number. Define a function $f(x) = a \sin x - x \sin^2 a$ such that $f(0) = f(a) = 0$. By Rolle's theorem there exists a number k lying in the open interval $(0, 1)$ such that $f'(ka) = 0$. What is the value of k if $a \to 0$?

(a) 1/4 (b) 1/2

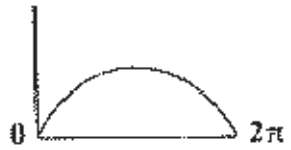
(c) 3/4 (d) 3/8

48. What is the value of $\int_0^{n\pi} \left|1 + \sin\left(\frac{x}{n}\right)\right| dx$ where $n \in I$?

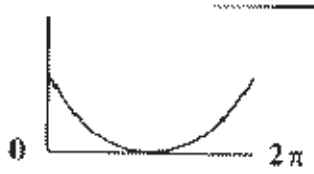
(a) n (b) $2n$

(c) $3n$ (d) $4n$

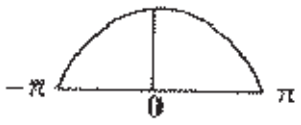
49. Match List-I (Curve) with List-II (Trace) and select the correct answer using the codes given below the lists :

List - I	List - II
A. $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$	1. 
B. $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$	2. 

C. $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

3. 

D. $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

4. 

	A	B	C	D		A	B	C	D
(a)	4	2	3	1	(b)	1	3	2	4
(c)	4	3	2	1	(d)	1	2	3	4

50. Two rods OA, OB are rigidly fixed at O at right angles to each other and they rotate about O in their plane with constant angular velocity ω. What are the accelerations components of a particle whose coordinates with respect to OA, OB are for all time (α, β) where α, β are constants ?

(a) $(-\omega^2\alpha, \omega^2\beta)$ along OA and OB respectively

(b) $(\omega^2\alpha, \omega^2\beta)$ along OA and OB respectively

(c) $(\omega^2\beta, \omega^2\alpha)$ along OA and OB respectively

(d) $(\omega^2\beta^2/\alpha, \omega^2\alpha^2/\beta)$ along OA and OB respectively

51. A satellite is launched in a direction parallel to the surface of the earth with a velocity of 18,820 miles/hr from an altitude of 240 miles. If the satellite reaches its maximum altitude of 2340 miles, what is the approximate velocity of the satellite (radius of earth = 3960 miles) ?

(a) 11,000 miles/hr (b) 12,550 miles/hr

(c) 16,550 miles/hr (d) 14,550 miles/hr

52. Which one of the following number systems results in the largest storage requirements ?

(a) Hexadecimal System (b) Octal System

(c) Decimal System (d) Binary System

53. What is the maximum number of different characters that an 8-bit code represents ?

(a) 16 (b) 64
(c) 128 (d) 256

Directions **: The following 5 (FIVE) items consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below:**

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is ***not*** the correct explanation of A

(c) A is true but R is false (d) A is false but R is true

54. **Assertion (A):** If θ_1 and θ_2 (where $\theta_1 < \theta_2$) are the two angles of projection of a particle projected with velocity u, and R_1 and R_2 are the corresponding ranges, then R_1 must be less than R_2.

Reason (R) : The range of a projectile is given by $R = (u^2 \sin 2\theta)/g$ where θ is the angle of projection.

55. **Assertion (A):** The set of all real polynomials P is a vector space.

Reason (R) : The linearly independent set $\{1, x, x^2, ..., x^n\}$ generates P.

56. **Assertion (A):** The set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1$ is a subgroup under matrices multiplication.

Reason (R) : The set of all 2×2 real matrices forms a group under matrix multiplication.

57. **Assertion (A):** The storage devices are both input and output devices.

Reason (R) : The programs, during execution, can read data

stored in auxiliary memory for input and write the output of the program into the auxiliary memory.

58. **Assertion (A)** : The set of vectors (1, 0, 1), (0, 1, 1) and (1, 1, 1) generates the vector space R^3.

Reason (R) : The given set of vectors forms a standard basis.

59. If a, b, c are the roots of the equation $x^3 - px^2 + qx - r = 0$, what is the value of $(a + b)(b + c)(c + a)$?

(a) $pq - r$ (b) $pq + r$

(c) pqr (d) pq/r

60. What is the dimension of the vector space formed by the solutions of the system of the following equations ?

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - 2x_2 = 0$$
$$x_2 - x_3 = 0$$

(a) 1 (b) 2

(c) 3 (d) 0

61. Let $T : R^2 \to R^2$ be defined by $T(x, y) = (x + y, x - y)$. Which one of the following is the matrix of T for (0, 1) and (1, 0) as a basis for both domain and range of T ?

(a) $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

62. Let $T : R^2 \to R^2$ be a map defined by $T(x, y) = (x + y, x - y)$. Which one of the following statements is correct ?

(a) T is linear and its kernel has infinite number of elements of R^2

(b) T is not linear

(c) The kernel of T consists of only two elements of R^2

(d) Nullity of T is zero

63. Let $T : R^2 \to R^2$ be a linear transformation such that

$$T\left(\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\2\end{pmatrix} \text{ and } T\left(\begin{pmatrix}0\\1\end{pmatrix}\right) = \begin{pmatrix}2\\1\end{pmatrix}$$

What is $T\left(\begin{pmatrix}1\\1\end{pmatrix}\right)$ equal to ?

(a) $\begin{pmatrix}1\\2\end{pmatrix}$ (b) $\begin{pmatrix}2\\1\end{pmatrix}$

(c) $\begin{pmatrix}3\\3\end{pmatrix}$ (d) $\begin{pmatrix}2\\2\end{pmatrix}$

64. What is the output of the following algorithm ?

Step 1 : get n, r

Step 2 : k = 1

Step 3 : p = 1

Step 4 : t = n - [illegible] + k

Step 5 : p = [illegible] t

Step 6 : k = [illegible]

Step 7 : If k < [illegible] + 1

then go to step 4

else go to step 8

Step 8 : Write p

Step 9 : Stop

(a) $\lfloor n$

(b) $\lfloor n \quad r$

(c) $\frac{\lfloor n}{\lfloor n - r}$

(d) $\frac{\lfloor n}{\lfloor n - r + 1}$

65. Given the vectors $\alpha = (1, 2, 3)$, $\beta = (3, 1, 0)$, $\gamma = (2, 1, 3)$ and $\delta = (-1, 3, 6)$.

Consider the following statements :

1. γ is a linear combination of α and β.
2. δ is a linear combination of α and β.

Which of the following statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

66. What is the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

67. Let A and B be any two $n \times n$ matrices and

$$\text{tr } A = \sum_{i=1}^{n} a_{ii} \text{ and tr } B = \sum_{i=1}^{n} b_{ii}$$

Consider the following statements :

1. tr (AB) tr (BA)
2. tr (A + B) tr A tr B

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

68. Let $A = \begin{pmatrix} 2 & 0 \\ 3 & -5 \end{pmatrix}$ be expressed as P + Q where P is symmetric matrix and Q is skew-symmetric matrix. Which one of the following is correct ?

(a) $Q = \begin{pmatrix} 1/2 & -3/2 \\ 3/2 & 0 \end{pmatrix}$ (b) $Q = \begin{pmatrix} 0 & 3/2 \\ 3/2 & 0 \end{pmatrix}$

(c) $Q = \frac{1}{2}\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$ (d) $Q = \frac{1}{2}\begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}$

69. What is the determinant of the following matrix ?

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & \cdots & 0 \\ 1 & 1/2 & 0 & \cdots & \cdots & 0 \\ 1 & 1/2 & 1/3 & \cdots & \cdots & 0 \\ 1 & 1/2 & 1/3 & 1/4 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 1/2 & 1/3 & 1/4 & \cdots & 1/n \end{pmatrix}$$

(a) $n^2 + n^2 + 1$ (b) $\frac{1}{n^2} + \frac{1}{n} + 1$

(c) $\frac{1}{\lfloor n}$ (d) $\frac{n(n+1)}{2}$

70. If the system of equations
$x + 2y - 3z = 1$, $(p + 2)z = 3$, $(2p + 1)y + z = 2$
is inconsistent, then what is the value of p ?
(a) 2 (b) $-1/2$
(c) 0 (d) 2

71. If the equation $x^3 + ax^2y + bxy^2 + y^3 = 0$
represents three lines, two of which are perpendicular to each other, then what is the equation of the third line ?
(a) $y = x$ (b) $y = bx$
(c) $y = -x$ (d) $y = -x$

72. A straight line is defined by the intersection of a plane parallel to yz plane and a plane through x-axis. If $p \neq 0$, then what is the possible form of direction ratios of this straight line ?
(a) (0, p, 1) (b) (p, 0, 1)
(c) (1, 0, p) (d) (1, p, 0)

73. For what values of A and B, are the two planes represented by the equation $x^2 + y^2 + Az^2 + 2B(yz + zx + xy) = 0$

orthogonal ?

(a) A = 1, B = 1 (b) A = –2, B = 1

(c) A = 2, B = –1 (d) A = –1, B = 2

74. What is the maximum distance from the point (1, 2, –1) to the surface of the sphere $x^2 + y^2 + z^2 = 24$?

(a) $\sqrt{6}$ (b) $2\sqrt{6}$

(c) $3\sqrt{6}$ (d) $4\sqrt{6}$

75. For what value(s) of a, will the two points (1, a, 1) and (–3, 0, a) lie on opposite sides of the plane

$3x + 4y - 12z + 13 = 0$?

(a) $a < -1$ or $a > 1/3$ (b) a = 0 only

(c) $0 < a < 1$ (d) $-1 < a < 1$

76. Three vectors of magnitudes 1, 2, 3 units meeting at the corner of a cube are directed along the diagonals of the three faces meeting at the corner. What is the magnitude of their resultant ?

(a) 4 units (b) 5 units

(c) 6 units (d) 7 units

77. OABC is a parallelogram with $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$. What is the expression of $\vec{a} + 1/2$ ($\vec{c} - \vec{a}$) equal to ?

(a) $\vec{b}$ (b) $\vec{b}/2$

(c) $(\vec{b} - \vec{a})/2$ (d) $\vec{c}$

78. ABCDEF is a regular hexagon.

Consider the following :

1. $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD}$
2. $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3(\overrightarrow{AE} + \overrightarrow{AB})$
3. $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 2(\overrightarrow{AC} + \overrightarrow{AE})$

Which of the equations given above are correct ?

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2 and 3

79. If $\vec{p} = \vec{b} \times \vec{c}$, $\vec{q} = \vec{c} \times \vec{a}$ and $\vec{r} = \vec{a} \times \vec{b}$ are three vectors, then which one of the following is correct ?

(a) $\vec{a}$ is parallel to $\vec{q} \times \vec{r}$

(b) $\vec{a}$ is perpendicular to $\vec{q} \times \vec{r}$

(c) $\vec{a}$ is parallel to $\vec{r} \times \vec{p}$

(d) $\vec{a}$ is perpendicular to $\vec{r} \times \vec{p}$

80. $\vec{b} \times (\vec{b} \times \vec{a})$ is coplanar with

(a) $\vec{a}$ only (b) $\vec{b}$ only

(c) both $\vec{a}$ and $\vec{b}$ (d) neither $\vec{a}$ nor $\vec{b}$

81. If the vectors $\overrightarrow{AB} = -3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, thant what is the length of the median through A ?

(a) $\sqrt{14}$ (b) $\sqrt{18}$

(C) $\sqrt{29}$ (D) $\sqrt{7}$

82. If $\vec{a}$, $\vec{b}$ are unit vectors such that the vector $\vec{a} + 3\vec{b}$ is perpendicular to $7\vec{a} - 5\vec{b}$, what is the angle between $\vec{a}$ and $\vec{b}$?

(a) $\pi/2$ (b) $\pi/6$

(c) $\pi/3$ (d) π

83. Given $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{B} \times \vec{C} = \vec{A}$.
Consider the following :

1. $\vec{A}$, $\vec{B}$, $\vec{C}$ are mutually orthogonal.
2. $|\vec{A}| = |\vec{C}|$
3. $|\vec{B}| = 1$

Select the correct answer using the codes given below :

(a) 1, 2 and 3 (b) 1 and 2

(c) 2 and 3 (d) 1 and 3

84. Which one of the following differential equations has the sum of order and degree ?

(a) $\frac{d^4y}{dx^4} + 8\left(\frac{dy}{dx}\right)^4 + 5y = e^x$

(b) $5\left(\frac{d^3y}{dx^3}\right)^4 + 8\left(\frac{dy}{dx} + 1\right)^2 + 5y = x^3$

(c) $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{2/3} = 4\left(\frac{d^3y}{dx^3}\right)$

(d) $y = x^2\left(\frac{dy}{dx}\right) + \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}$

85. Which of the following are the orthogonal trajectories of the curves $A = r^2 \cos\theta$?

(a) $B = r \sin^2\theta$ (b) $B = r^2 \sin\theta$

(c) $B = r^3 \sin^2\theta$ (d) $B = r^3 \sin\theta$

86. What is the solution of the differential equation

$\sin px \cos y = \cos px \sin y + p$

(a) $y = cx^2 - \sin^{-1} c$ (b) $y = cx - \sin^{-1} c$

(c) $x = cy - \sin^{-1} c$ (d) $x = cy^2 - \sin^{-1} c$

87. What is the general solution of the differential equation

$(D^2 - 3D + 2)\, y = \sinh x$?

(a) $y = c_1e^x + c_2e^{2x} - \frac{xe^x}{2} + \frac{e^{-x}}{12}$

(b) $y = c_1e^x + c_2e^{2x} - \frac{xe^x}{2} + \frac{xe^{-x}}{12}$

(c) $y = c_1e^x + c_2e^{2x} + \frac{e^x}{2} + \frac{xe^{-x}}{12}$

(d) $y = c_1e^x + c_2e^{2x} - \frac{e^x}{2} + \frac{xe^x}{12}$

88. Curves passing through the point (0, 1) and satisfying the equation $\sin\left(\frac{dy}{dx}\right) = c$ are given by

(a) $\cos\left(\frac{y+1}{x}\right) = c$ (b) $\cos\left(\frac{x}{y+1}\right) = c$

(c) $\sin\left(\frac{y-1}{x}\right) = c$ (d) $\sin\left(\frac{x}{y+1}\right) = c$

89. What is the singular solution of the differential equation $p = \ln(px - y)$?

(a) $y = cx - e^c$ (b) $y = x + \ln x$

(c) $y = x \ln x - x$ (d) $y = x \ln x - c$

90. The differential equation $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ is reduced to linear form $\frac{dv}{dx} + Pv = Q$ where P and Q are function of x alone, by changing the variable as

(a) $\sin y = v$ (b) $\cos y = v$

(c) $\tan y = v$ (d) $\sin 2y = v$

91. What is the solution of the differential equation $x\left(\frac{dy}{dx}\right) = y(\ln y - \ln x + 1)$?

(a) $y = xe^{cx}$ (b) $y + xe^{cx} = 0$

(c) $y + ce^x = 0$ (d) $x = e^{cx}$

92. Let y_1 and y_2 be any two solutions of a second-order linear non-homogenous ordinary differential equation and c be an

arbitrary constant. Then in general

(a) $y_1 + y_2$ is its solution, but cy_1 is not

(b) cy_1 is its solution, but $y_1 + y_2$ is not

(c) both $y_1 + y_2$ and cy_1 are its solutions

(d) neither $y_1 + y_2$ nor cy_1 is its solution

93. Consider the following statements regarding the differential equation

$$\left|\frac{dy}{dx}\right| + |y| = 0,\ 0 < x < 1$$

satisfying $y(0) = 1$:

1. It is a linear differential equation.
2. It has a unique solution.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

94. What is the value of the integral $\int_{-1}^{1} \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx$?

(a) $x \ln (2 + x\cos x)$ (b) $x \ln (2 - x\cos x)$

(c) $x\cos x$ (d) 0

95. An aperture is in the form of a rectangle surmounted by an equilateral triangle. The height of the rectangle is b and the side of the equilateral triable is a. The total perimeter of the aperture is fixed. Under what condition will the aperture admit maximum amount of light ?

(a) $2b = (3 + \sqrt{3})a$ (b) $2b = (\sqrt{3} + 1)a$

(c) $2b = (3 - \sqrt{3})a$ (d) $2b = (\sqrt{3} - 1)a$

96. If $\int_a^b f(x)\,dx = I, \int_{ac}^{bc} f(x/c)\,dx,\ c \neq 0$

then what is the value of λ ?

(a) $b - a$ (b) $a - b$

(c) c (d) $1/c$

97. What is the value of the integral $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$?

(a) 1/4 (b) 1/3

(c) 1/2 (d) 1

98. What is the length of an arc of the curve $y = |x - 1| + |x - 2|$ from $x = 1$ to $x = 3$?

(a) 1/3 (b) 1/2

(c) 2 (d) 3

99. What are the asymptotes of the curve $(x + 4y + 1)(x - 4y + 1) = 4$?

(a) $y = \pm 4(x + 1)$ (b) $y = \pm 4x$

(c) $x = \pm 4y$ (d) $x + 1 = \pm 4y$

100. At what point on the curve $y = [(e^x + e^{-x})/2]^2$, is the curvature maximum ?

(a) (1/2, 1/2) (b) (0, 1)

(c) (2, 0) (d) (0, 1/2)