### 8.1 Alternating Voltage and Alternating Current (A. C.)

The following figure shows N turns of a coil of conducting wire PQRS rotating with a uniform angular speed $\omega$ with respect to the X -axis in a uniform magnetic field along the Y -axis.


Let the angle between the direction of the arew vector of the coil and the magnetic field direction be zero at time, $t=0$ and $\left(=\omega t\right.$ at time $t=t$. The magnetic flux, $\Phi_{0}$, associated with the coil at time $t=0$ and $\Phi$. at 'me $t=t$ are given by
$\Phi_{0}=\mathbf{N} \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=\mathbf{N A B} \cos 0=N, \quad$ and
$\Phi_{t}=N A B \cos \omega t$
The emf induced in t e oi' according to Faraday's law is
$V=-\frac{d \Phi_{t}}{d t}=A^{\prime} A B \sin \omega t=V_{m} \sin \omega t \quad \ldots \quad$... $\ldots$...
where $V_{m}=$ N $A B$ is the maximum value of the induced emf.
Eru. 'in' (i) shows that the induced emf versus time is a sine curve. This emf is obtained be, veen de brushes $B_{1}$ and $B_{2}$ which are in contact with the slip rings $A_{1}$ and $A_{2}$ as s. $n, n$ in the above figure.
I., e voltage is zero at time $t=0$ and varies as per the function $\sin \omega t$ reaching maximum value $V_{m}$ at time $t=\pi / 2 \omega$ and again zero at time $t=\pi / \omega$. $B_{2}$ being at greater potential than $B_{1}$ acts like a positive end of the voltage source during this time interval.

After time $t=\pi / \omega$, the potential of $B_{1}$ starts to rise with respect to $B_{2}$ till time $t=3 \pi / 2 \omega$, reaches maximum in the reverse direction and again becomes zero at time $t=2 \pi / \omega$. This cycle keeps repeating in every time interval of $T=2 \pi / \omega$.

The voltage so developed is known as alternating voltage and its graph versus time is shown by a continuous line in the following figure. The arrangement to produce such a voltage is
known as A.C. (alternating current) dynamo or generator.


In the adjoining figure, an alternating voltage source is connected in a circuit with resistor, $R$. The voltage between $a$ and $b$ is zero at time $\mathbf{t}=\mathbf{0}$. Zero current will flow at this time.

The voltage in the circuit varies according as $\mathbf{V}=\mathbf{V}_{\mathbf{m}} \sin \omega \mathrm{t}$. The current as per Ohm's law will be $I=\frac{V_{m} \sin \omega t}{R}$. The current chi ages
 sinusoidally in the same way as the $v$ 'эge as shown by the broken line in the above ، rap.

Here, voltage was considered changı. $\mathrm{I}_{\mathrm{w} . \mathrm{il}}$ time as per $\sin \omega \mathrm{t}$. Both the voltage and the current can be considered to $b^{\prime}$ cr. ngi. $j$ as per cos $\omega$ t. It is not necessary that the voltage and current should change is at ove. There are also other ways in which both can change periodically with time.

### 8.2 A. C. Circuit win Ser 3 : Combination of Resistor, Inductor and Capacitor

The following figure s owe a series combination of resistor having resistance $R$, inductor of inductance $L$ ar. I a capacitor of capacitance $C$ with an alternating voltage source of voltage changing wit' tin ? as $V=V_{m} \cos \omega \mathrm{t}$. It is assumed that the resistor has zero inductance and the infuc. or : s zero resistance.

Let I - c rrent in the circuit,
Q. $\therefore$ rge on the capacitor and $\frac{a_{1}}{{ }^{\prime} t}=$ ate of change of current at inls t.

Hence, by Kirchhoff's law,

$$
V_{R}+V_{L}+V_{C}=V
$$

$\therefore I R+L \frac{d I}{d t}+\frac{Q}{C}=V_{m} \cos \omega t$
Putting $I=\frac{d Q}{d t}$ and $\frac{d I}{d t}=\frac{d^{2} Q}{d t^{2}}$ in the above equation,
$R \frac{d Q}{d t}+L \frac{d^{2} Q}{d t^{2}}+\frac{Q}{C}=V_{m} \cos \omega t$
$\therefore \frac{d^{2} Q}{d t^{2}}+\frac{R}{L} \frac{d Q}{d t}+\frac{Q}{L C}=\frac{V_{m}}{L} \cos \omega t$
This equation is known as the differential equation of $Q$ for the series $R-L-{ }^{\text {r }}$, C. 'ircuit. It is similar to the differential equation of forced harmonic oscillations given : un ?r.

$$
\frac{d^{2} y}{d t^{2}}+\frac{b}{m} \frac{d y}{d t}+\frac{k}{m} y=\frac{F_{0}}{m} \sin \omega t
$$

The mechanical and electrical quantities in the above equations are $\quad \boldsymbol{m}$ ared in the following table.

| No. | Mechanical quantities | Symbol | Eler .rical 4. .cities | Symbol |
| :---: | :---: | :---: | :---: | :---: |
| 1. | Displacement | y | - 'cti. ${ }^{\text {n' }}$ charge | Q |
| 2. | Velocity | $\frac{d y}{d t}$ | ler ric current | $\frac{d Q}{d t}=I$ |
| 3. | Co-efficient of resistance | b | Resistance | R |
| 4. | Mass | n. | Inductance | L |
| 5. | Force constant | k | Inverse of capacitance | 1 |
| 6. | Angular frequency | $\sqrt{\frac{k}{m}}$ | Angular frequency | $\frac{1}{\sqrt{\text { LC }}}$ |
| 7. | Periodic force | $\mathrm{F}_{0}$ | Periodic voltage | $\mathrm{V}_{\mathrm{m}}$ |

The function of shares $Q$ versus time $t$ which satisfies the above differential equation of charge $\mathbf{Q}$ is . 'le.' its solution. Complex function is used to find the solution of the above differentia! $\quad$ qu tic.

### 8.3 C. $\quad$ - 'ex Numbers (For Information Only)

A omp. $x$ number $z=x+j y$, where $j=\sqrt{-1} . x$ is the real part and $y$ is the imaginary $\cdots$ of the complex number.

1) The complex number can be represented by a point in a complex plane with x-axis representing real numbers and $y$-axis representing imaginary numbers. Point $P$ in the figure (next page) represents the complex number $z=x+j y$. The $x$-coordinate of $P$ gives the real part of $z$ and $y$-coordinate gives its imaginary part. The magnitude of complex number is equal to $r$, i.e., $|z|=r=\sqrt{x^{2}+y^{2}}$.

Now, $x=r \cos \theta$ and $y=r \sin \theta$
$\therefore z=r \cos \theta+j r \sin \theta$
$=r(\cos \theta+j \sin \theta)$
$=\mid z I e^{j \theta} \quad\left(\because e^{j \theta}=\cos \theta+j \sin \theta\right)$
(2) $z^{*}=x-j y$ is the complex conjugate of the complex number $z$.
(3) For complex number $z$,

$$
\begin{aligned}
\frac{1}{z} & =\frac{1}{z} \frac{z^{*}}{z^{*}}=\frac{z^{*}}{|z|^{2}}=\frac{x-j y}{x^{2}+y^{2}} \\
& =\frac{x}{x^{2}+y^{2}}-j \frac{y}{x^{2}+y^{2}}
\end{aligned}
$$



The real part of $\frac{1}{z}=\frac{x}{x^{2}+y^{2}}$ and the imag ،ary Du.. of $\frac{1}{z}=-j \frac{y}{x^{2}+y^{2}}$.
The real part of the complex number $z$ is ler $s t+$ by $\operatorname{Re}(z)$ and the imaginary part by $\operatorname{Im}(z)$.

### 8.4 Solution of the differential equ atil 7

The equation for series $L-C-R$ cir ${ }^{\text {t }}$ is $I R+L \frac{d I}{d t}+\frac{Q}{C}=V_{m} \cos \omega t$
$\therefore \quad I R+L \frac{d I}{d t}+\frac{\int I d t}{C} \quad V_{n .} r s \omega t$ where $Q=\int I d t$
$\therefore \quad \frac{d I}{d t}+\frac{R}{L} I+\frac{1}{L t} \int r^{\prime} d t=\frac{V_{m}}{L} \cos \omega t$
Replacing the c rrent $I$ by complex current $i$ and $\cos \omega t$ by $e^{j \omega t}$, and solving the differential ec. . ${ }^{\text {ic }}$, of complex quantities, the real part of complex current i will give the equation $c$. ${ }^{2} e_{i}{ }^{\prime} c_{c}$ rent $I$ as a function of time, $t$.
Thus, . , ifferential equation of complex quantities is
$\frac{d}{1+}+\frac{-}{L} t+\frac{1}{L C} \int i d t=\frac{V_{m}}{L} e^{j \omega t} \quad \ldots \ldots(1), \quad$ where $R, L, C$ and $t$ are real quantities.
Li $i=i_{m} \mathrm{e}^{\mathrm{j} \omega \mathrm{t}}$ be a trial solution.
$\therefore \quad \frac{d i}{d t}=i_{m} \cdot j \omega \cdot e^{j \omega t} \quad$ and $\quad \int I d t=\frac{i_{m} e^{j \omega t}}{j \omega}$
Putting the values of $i, \frac{\mathrm{~d} i}{\mathrm{dt}}$ and $\int \mathrm{Idt}$ in equation (1) above, we have $i_{m} \cdot j \omega \cdot \mathrm{e}^{j \omega t}+\frac{R}{\mathrm{~L}} i_{m} \mathrm{e}^{j \omega t}+\frac{1}{L C} \frac{i_{m} \mathrm{e}^{j \omega t}}{j \omega}=\frac{V_{m}}{L} \mathrm{e}^{j \omega t}$
$\therefore \quad i_{m}\left(j \omega+\frac{R}{L}+\frac{1}{j \omega L C}\right)=\frac{V_{m}}{L}$
$\therefore \quad i_{m}\left(j \omega L+R+\frac{1}{j \omega C}\right)=V_{m}$
$\therefore \quad i_{m}\left(j \omega L+R-\frac{j}{\omega C}\right)=v_{m} \quad\left(\because \frac{1}{j}=\frac{j}{j^{2}}=-j\right)$
$\therefore \quad i_{m}=\frac{V_{m}}{R+j\left(\omega L-\frac{1}{\omega C}\right)}$
Putting this value of $i_{m}$ in the trial solution, we have
$i=\frac{V_{m} e^{j \omega t}}{R+j\left(\omega L-\frac{1}{\omega C}\right)}$
This equation shows that the resistance off ied $\mathrm{y} \|$ inductor and a capacitor are $\mathrm{j} \omega \mathrm{L}$ and $-\mathrm{j} / \omega \mathrm{C}$ which are known as inductive gacta...ce and capacitive reactance respectively. Their magnitudes are $\omega \mathrm{L}$ and $1 / \omega \mathrm{C}$ re ipe 'ive.: The inductive and capacitive reactance are represented by symbols $Z_{L}$ and $Z_{c} w$. ile thir magnitudes are equal to $X_{L}$ and $X_{c}$.
$Z_{L}=j \omega L, \quad Z_{c}=-j / \omega r$,
$X_{L}=\omega L, \quad X_{c}=1^{\prime} a s$.
The summation of $Z_{L}, Z_{c} \ldots+R$ is called the impedance $(Z)$ of the series L-C-R circuit. The unit of impedance is $\mathfrak{v m}$
$\therefore \mathbf{Z}=\mathbf{R}+\mathbf{Z}_{\mathrm{L}}+\cdots \cdots \mathbf{R}+\mathrm{j}(\omega \mathrm{L}-\mathbf{1} / \omega \mathrm{C})$
$\therefore i=\frac{\mathrm{V}_{\mathrm{m}}}{\mathrm{T}}-\mathrm{ot}=\frac{\mathrm{V}}{\mathrm{Z}}$
The i 'n't equation represents Ohm's law for complex current, complex voltage and ir. $\operatorname{je} \cdot \boldsymbol{n}$ e. impedance is also complex which can be expressed as $Z=|z| e^{j \delta}$.
$\because t=\frac{V_{m} e^{j \omega t}}{|Z| e^{j \delta}}=\frac{V}{|Z|} e^{j(\omega t-\delta)}=\frac{V}{|Z|}[\cos (\omega t-\delta)+j \sin (\omega t-\delta)]$
where, $|z|=\sqrt{R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}$
Now, $I=\operatorname{Re}(i)=\frac{V_{m} \cos (\omega t-\delta)}{\sqrt{R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}}=\frac{V_{m} \cos (\omega t-\delta)}{|Z|}$

The equation shows that the current in the circuit changes according to $\cos (\omega \mathrm{t}-\boldsymbol{\delta})$ and lags the voltage by phase $\delta$ as shown in the following graph.



In the second graph above, point $H$ shows the comp $x$ im edance, $Z=R+j(\omega L-1 / \omega C)$ in the complex plane where $x$-coordinate of the noi, is $R$ which is the real part of the complex impedance and y-coordinate of the poir is $n \mathrm{~L}-1 / \omega \mathrm{C}$ which is the imaginary part of the complex impedance.
From the figure, $\tan \delta=\frac{H D}{O D}=\frac{\omega L-\frac{1}{a}}{a}$
and $\quad|\mathrm{Z}|=\mathrm{OH}=\cdot \sqrt{(\ldots 7)^{2}} \overline{(\mathrm{DH})^{2}}=\sqrt{R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}$

### 8.5 Different Cases

The rules regarding the -quivi lent resistances for series and parallel connection of $\mathbf{R}$ are also applicable to $j \omega \mathrm{~L}$ a $1 \mathrm{~d}-{ }^{-} / \omega \mathrm{C}$. Using the above geometrical construction, the relationship between voltage and $\llcorner$ rrer for various circuits can be derived.
(1) A. C. Cire'it 'ontz ling only Inductor:
 showr in 'e flyure.



OA forms an angle $\pi / 2$ with the real axis indicating that the current lags the voltage by $\pi / 2$ as shown in the above figures. Further $O A=\omega L=|Z|$. So the equation for current can be written as
$I=\frac{V_{m} \cos \left(\omega t-\frac{\pi}{2}\right)}{\omega L}=\frac{V_{m} \cos \left(\omega t-\frac{\pi}{2}\right)}{X_{L}}$

## (2) A. C. Circuit containing only a capacitor:

The impedance $Z=-j / \omega C=-j X_{c}$ is represented by the point $r \ldots .$. ... $\quad \mathrm{mplex}$ plane as shown in the figure.


OF forms an angle $-\pi / 2$ witr thr reaı axis indicating that the current leads the voltage by $\pi / 2$ as shown in the above figu $\pm$. rurther $O F=1 / \omega C=|Z|$. So the equation for current can be written as
$I=\frac{V_{m} \cos \left(\omega t+\frac{\pi}{2},\right.}{\frac{1}{\triangle C}}=\frac{V_{m} \cos \left(\omega t+\frac{\pi}{2}\right)}{X_{C}}$
(3) A.C. C. ${ }^{1}$ : containing $R$ and $L$ in series:

The in $\nu_{c}$ ': ice $Z=R+j \omega L$ is represented by the $p$ int $d$ ' $\eta$ the complex plane as shown in the figure.

Th $|Z|=\sqrt{R^{2}+\omega^{2} L^{2}}=\sqrt{R^{2}+X_{L}^{2}}$
From the figure, $\delta=\tan ^{-1}\left[\frac{\omega L}{R}\right]=\tan ^{-1}\left[\frac{X_{L}}{R}\right]$

$$
I=\frac{V_{m} \cos (\omega t-\delta)}{\sqrt{R^{2}+\omega^{2} L^{2}}}
$$

Here the current lags the voltage by a phase $\delta$, as given by the above equation for $\delta$.

(4) A. C. circuit with $R$ and $C$ connected in series:

The impedance $Z=R-j / \omega C=R-j X_{C}$ is represented by the point $H$ in the complex plane as shown in the figure.
$\mathrm{OH}=|\mathrm{Z}|=\sqrt{\mathrm{R}^{2}+\frac{1}{\omega^{2} C^{2}}}=\sqrt{\mathrm{R}^{2}+X_{C}{ }^{2}}$
Here, $\delta$ is negative and is given by
$\delta=\tan ^{-1}\left[\frac{1}{\omega C R}\right]=\tan ^{-1}\left[\frac{X_{C}}{R}\right]$ and
$I=\frac{V_{m} \cos (\omega t+\delta)}{\sqrt{R^{2}+\frac{1}{\omega^{2} C^{2}}}}=\frac{V_{m} \cos (\omega t+\delta)}{\sqrt{R^{2}+X_{C}{ }^{2}}}$
Here the current leads the voltage by a phase $\delta$, . gisn' y the above equation for $\delta$.
(5) An A. C. circuit having L and C connecte in s...es:

The impedance $Z=j \omega L-j / \omega C \quad X_{L} \quad j X_{C}$ is represented by the point $G$ in the omp. $\times$ plane as shown in the figure.
$|z|=\omega L-1 / \omega C=X_{L}-y ;$
Here $\omega \mathrm{L}>1 / \omega \mathrm{C}$, and c ricit lags voltage by a phase $\pi / 2$.
$\therefore \delta=\pi / 2$ ind $\quad=\frac{v_{m} \cos \left(\omega t-\frac{\pi}{2}\right)}{\omega L-\frac{1}{\omega C}}$
If $\omega \mathrm{L}<1$, , current leads voltage by a phase $\pi / 2$ in
$C=-\pi / 2$ and $I=\frac{V_{m} \cos \left(\omega t+\frac{\pi}{2}\right)}{\omega L-\frac{1}{\omega C}}$
(6) A. C. circuit containing parallel combination of $L$ and $C$, connected in series with $R$ :

If $Z_{1}=$ resultant impedance of the parallel combination of the inductor and the capacitor,
$\frac{1}{Z_{1}}=\frac{1}{Z_{C}}+\frac{1}{Z_{L}}=\frac{1}{-\frac{j}{\omega C}}+\frac{1}{j \omega L}=j\left[\omega C-\frac{1}{\omega L}\right]$
$\therefore Z_{1}=\frac{1}{j\left[\omega C-\frac{1}{\omega L}\right]}=-\frac{j}{\left[\omega C-\frac{1}{\omega L}\right]}$
$\therefore$ equivalent impedance of the circuit,
$Z=Z_{1}+R=R-\frac{j}{\left[\omega C-\frac{1}{\omega L}\right]}$
If $\omega C>\frac{1}{\omega L}$, the impedance $Z$ can be represented in the complex plane as shown in the graph.

Here, $\delta$ is negative and given by
$\tan \delta=\frac{H D}{O D}=\frac{1}{R\left[\omega C-\frac{1}{\omega L}\right]}$

$|Z|=\sqrt{R^{2}+\frac{1}{\left[\omega C-\frac{1}{\omega L}\right]^{2}}} \quad a n^{\prime} \quad I=\frac{V_{m} \cos (\omega t+\delta)}{\sqrt{R^{2}+\frac{1}{\left[\omega C-\frac{1}{\omega L}\right]^{2}}}}$

## 8.6 r.m.s. Values of Alw. $-n+i n g$ Voltage and Current

Special ammeters an $v^{\prime}$ rretars are used to measure alternating voltage and current. These meters measure the r. .s. 1 soot mean square) value of the voltage and current.

Root mean srua, , of , quantity varying periodically with time means the square root of the mean of the ${ }^{\prime} 1^{9} \boldsymbol{\prime}$ 's of the quantity, taken over a time equal to the periodic time.

For $V=1{ }_{\eta} \cos \omega t$,
th. av. Je value of $v^{2} \equiv\left\langle v^{2}\right\rangle=\left\langle V_{m}{ }^{2} \cos ^{2} \omega t\right\rangle$

$$
=V_{m}^{2}\left\langle\frac{1+\cos 2 \omega t}{2}\right\rangle=V_{m}^{2}\left\langle\frac{1}{2}+\frac{\cos 2 \omega t}{2}\right\rangle
$$

The average value of $1 / 2$ is $1 / 2$ and that of $\cos 2 \omega t$ is zero for one periodic time.

$$
\begin{aligned}
& \left.\therefore<V^{2}\right\rangle=\frac{V_{m}^{2}}{2} \\
& \therefore \quad V_{\text {r.m.s. }}=\sqrt{\left\langle V^{2}\right\rangle}=\frac{V_{m}}{\sqrt{2}} \quad \text { Similarly, } \quad I_{\text {r.m.s. }}=\frac{I_{m}}{\sqrt{2}}
\end{aligned}
$$

### 8.7 Series Resonance

The current in $L-C-R$ series circuit is given by

$$
\begin{aligned}
I & =\frac{V_{m} \cos (\omega t-\delta)}{\sqrt{R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}} \\
\therefore \quad I & =I_{m} \cos (\omega t-\delta), \quad \text { where, } \quad I_{m}=\frac{V_{m}}{\sqrt{R^{2}+\left(\omega L-\frac{1}{a^{2}}\right)^{2}}}
\end{aligned}
$$

$$
\text { Now, } I_{\text {r.m.s. }}=\frac{I_{m}}{\sqrt{2}}=\frac{\frac{V_{m}}{\sqrt{2}}}{\sqrt{R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}}=\frac{V . . \mathrm{s} \text { s. }}{V^{Z^{2}}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}=\frac{\text { Vr.m.s. }}{|Z|}
$$

Thus, value of $I_{\text {r.m.s. }}$ varies with $\omega$. If $\omega-\omega_{0} i$ : ts sen such that $\omega_{0} L=1 / \omega_{0} C$, then $I_{\text {r.m.s. }}=\frac{V \text { r.m.s. }}{R}$ which is the maxir'm $V_{c}$ 'נe of $I_{\text {r.m.s. }}$

The L-C-R series circuit is da` to 'e in resonance when the r.m.s. value of the current becomes maximum for a part'cl،ar fre, eency, $\omega_{0}$, of the voltage source.

Now, $\omega_{0}=\frac{1}{\sqrt{L C}}$ is lonown as the natural angular frequency or resonant angular frequency of the given $\mathrm{L}-\mathrm{C}-$, sert is circuit.

Resonance is ochip wed when the imaginary pai $\rightarrow$ 'he impedance is zero.

The $\mathrm{fi}_{1}$ : $\cdot$ hows the $I_{\text {r.m.s. }}$ versus $\omega$ い V/t for two different values of $\mathbf{R} \cdot \mathbf{R}_{1}$ - $\mathbf{R}_{\mathbf{2}}$ ). The resonance curve $w_{n} \cdot$ nes sharper as the value of $R$ Iduces.

## Q - Factor:

The sharpness of the resonance curve of the series $L-C-R \quad$ circuit is measured in terms of a quality known as the $Q$-factor.

The maximum power $I^{2}$ r.m.s. $R$ is
 obtained during resonance. The power
reduces to half of the maximum power when the $I_{\text {r.m.s. }}$ is equal to $\frac{I_{\text {r.m.s. }}(\max )}{\sqrt{2}}$. This situation is shown in the graph where it is found that there are two values of angular frequency, $\omega_{1}$ and $\omega_{2}$ for which power is reduced to half the maximum power.
$\left(\omega_{2}-\omega_{1}\right)=\Delta \omega$ is known as the half power bandwidth. The sharpness of ite 1 sonance increases with the decrease in the value of the half power bandwidth,

The $Q$ - factor is defined as $Q=\frac{\omega_{0}}{\Delta \omega}$.
Larger the value of $\mathbf{Q}$, sharper will be the resonance curve.
It can be proved that the value of $\Delta \omega=\frac{R}{L}$. Also we $k r, w w$ hat $\omega_{0}=\frac{1}{\sqrt{L C}}$.
$\therefore \quad Q=\frac{\omega_{0}}{\Delta \omega}=\frac{1}{R} \sqrt{\frac{L}{C}}$
Thus, the $\mathbf{Q}$ - factor depends on the compnn $\mathrm{nt}^{\prime}$ ( ${ }^{\text {' the circuit. It gives the information }}$ about the tuning of the circuit as well as thr sele 'ivi'y of the circuit.

To tune a known frequency source, like a - V. r a radio, one has to select the right value of the inductor or the capacitor. Rese ... ce 1 : obtained only when both L and C are present because at the time of resonance 'ey oncel reactance of each other. Hence, resonance never occurs in case of R-L or $\mathrm{P}^{\boldsymbol{n}}$. c . `uit.

### 8.8 Phasor Method

Consider a harmonic anc on $I=I_{m} \cos (\omega t+\delta)$.
A vector of mapnitude :... is drawn from the origin of a coordinate sys $m \tau^{-}$an angle of $\omega t+\delta$ with the X -axis as sho \%. i, the figure.

The phast $(\omega \mathrm{t} r \delta)$ changes with time, i.e., the angle betwee, $h$ vector shown in the figure and the X-axis krop c, cıanging with time. The vector rotates with the angurar frequency $\omega$ in the XY plane. Such a ru. ng vector is known as phasor.


The X-component of the vector at any instant $t$ gives the value of $I_{m} \cos (\omega t+\delta)$ which is the instantaneous value of the current. To add several functions like $I_{1} \cos \left(\omega t+\delta_{1}\right)$, $\mathrm{I}_{2} \cos \left(\omega \mathrm{t}+\delta_{2}\right)$, .... etc., one has to take the algebraic summation of the X -components of the respective phasors. One can deal with the sine function by taking the $\mathbf{Y}$-component of the vector.

Now, consider the example of addition of two harmonic functions,
$\mathrm{I}_{1}=\mathrm{I}_{1 \mathrm{~m}} \cos \left(\omega \mathrm{t}+\delta_{1}\right)$ and $\mathrm{I}_{2}=\mathrm{I}_{2 \mathrm{~m}} \cos \left(\omega \mathrm{t}+\delta_{2}\right)$.

The figure shows the vectors representing $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ and their resultant vector I. The amplitude of the resultant vector $=O P$ and its phase angle is $\phi$ using the law of triangle for addition of vectors. The magnitudes of $\mathrm{I}_{1}, \mathrm{I}_{2}$ and I are given by $I_{1 m}, \quad I_{2 m}$ and $I_{m}$ respectively.

The angle between $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ is $\delta_{2}-\delta_{1}$ as can be seen from the figure.

Now,

$$
\begin{aligned}
\mathrm{Im}^{2} & =\mathrm{I}_{1 m}^{2}+\mathrm{I}_{2 m}^{2} \\
& +2 \mathrm{I}_{1 m} \mathrm{I}_{2 m} \cos \left(\delta_{2}-\delta_{1}\right) \\
& =\mathrm{I}_{1 m}{ }^{2}+\mathrm{I}_{2 m}{ }^{2}+2 \mathrm{I}_{1 m} \mathrm{I}_{2 m} \cos \delta \\
& \text { where, } \delta=\delta_{2}-\delta_{1}
\end{aligned}
$$

### 8.9 Use of Phasor in an A.C. C: uit

An A.C. Circuit containing only resist :
In an A. C. circuit containing n ', : esistor, the phase difference between the voltag : an $c^{\prime}$ rrent is zero
( $\delta=0$ ). Here, current 1 is $\cdots$. $\quad \cdots / n$ along the X -direction and as $\delta=0$, voltage is lso drawn in the same direction as shown in the figu ?.

## An A. C. Circuit ontaining only inductor:

Here, curr nt • lu.; the voltage $V$ by $\pi / 2$ radian, or in other wor 's, l. voltage leads the current by $\pi / 2$ radian. Hence
I is rep est ied along the X -direction, V will be along the $Y$ - 'irec.' , as shown in the figure.

4 A. C. circuit containing only capacitor:
Here, current I leads the voltage V by $\pi / 2$ radian, or in other words, the voltage lags the current by $\pi / 2$ radian. Hence, if I is represented along the X -direction, V will be along the negative $Y$-direction as shown in the figure.


Circuit comprising only resistor


Circuit comprising only inductor


Circuit comprising only capacitor

L-C - R series A.C. circuit:
The phasor diagram of each of the component is shown in the figure.
Here, since L, C and R are in series, the same current flows through all the three components. If $V$ is the applied voltage, then
$\mathbf{V}=\mathrm{V}_{\mathrm{L}}+\mathrm{V}_{\mathrm{C}}+\mathrm{V}_{\mathrm{R}}$
where, $\mathrm{V}_{\mathrm{L}}, \mathrm{V}_{\mathrm{C}}$ and $\mathrm{V}_{\mathrm{R}}$ are the voltage drop across the inductor, the capacitor and the resistor respectively. If the current, I, flowing through the above circuit is represented along the
X-direction, then the phasor diagram of the series circuit will be as shown in the next figure.

It can be seen from the figure that
$\mathrm{V}^{2}=\left(\mathrm{V}_{\mathrm{L}}-\mathrm{V}_{\mathrm{C}}\right)^{2}+\mathrm{V}_{\mathrm{R}}{ }^{2}$
If $I_{m}$ is the maximum current, then
$V_{R}=I_{m} R, \quad V_{L}=I_{m} X_{L} \quad$ and $\quad V_{V}-I_{i} X_{C}$
$\therefore V^{2}=I_{m}{ }^{2}\left(X_{L}-X_{C}\right)^{2} \quad I_{n}{ }^{\prime} R^{\prime}$
$\therefore V=I m \sqrt{\left(X_{L}-X_{C}{ }^{2}+R^{2}\right.}$
For maximum $\mathbf{c}$ r rent, $v=\mathbf{V}_{\mathrm{m}}$
$\therefore \mathrm{I}_{\mathrm{m}}=\frac{=}{\sqrt{\left(1 . .-\mathrm{X}_{\mathrm{C}}\right)^{2}+\mathrm{R}^{2}}}=\frac{\mathrm{V}_{\mathrm{m}}}{\mid \mathrm{Z\mid}}$



If $\delta=\mu \cdot{ }^{-} e$ angle between the applied voltage and current phasors and $V_{L}>V_{C}$, then the $c_{l}$ re.. . gs the voltage by a phase $\delta$. If $\mathrm{V}_{\mathrm{L}}<\mathrm{V}_{\mathrm{C}}$, then current would have led the voltage.
$t_{1, ~(t)}$ the above graph,
$\tan \delta=\frac{V_{L}-V_{C}}{V_{R}}=\frac{I_{m} X_{L}-I_{m} X_{C}}{I_{m} R}=\frac{X_{L}-X_{C}}{R}$

### 8.10 L-C Oscillations

If the two ends of a charged capacitor are connected by a conducting wire or a resistor, then it gets discharged and the energy stored in the form of electric field between its two plates gets dissipated in the form of joule heat.

Now, consider an inductor having negligible resistance connected to a charged capacitor as shown in the figure. Such a circuit is called an L-C circuit.

Let $Q_{0}$ be the charge on the capacitor on the initially charged capacitor to which an inductor is connected.

Let $Q=$ charge on the capacitor at time $t=t$, and
I = circuit current during discharge of capacitor.
$\therefore$ applying Kirchhoff's law to the closed circuit, we have

$\therefore \quad-L \frac{d I}{d t}+\frac{Q}{C}=0$
But $I=-\frac{d Q}{d t} \quad$ ( negative sign indicates $t^{t}$ at he harge on the capacitor decreases with time.)
$\therefore L \frac{d^{2} Q}{d t^{2}}+\frac{Q}{C}=0$
$\therefore \frac{\mathrm{d}^{2} \mathrm{Q}}{\mathrm{dt}^{2}}=-\frac{\mathrm{Q}}{\mathrm{LC}}$
Comparing the above equation with ${ }^{4} \mathrm{e} a$. ${ }^{\text {© }}$ erential equation of the simple harmonic motion,
$\frac{d^{2} y}{d t^{2}}=-\omega_{0}{ }^{2} y$
$Q$ is analogous to the variad $=$, and $\omega_{0}{ }^{2}$ is analogous to $\frac{1}{L C}$. The solution to the above equation would be,
$Q=Q_{m} \sin \left(\omega_{l}+\phi\right) \quad \ldots \quad \ldots \quad \ldots \quad$ (1)
Here, $\mathbf{Q}_{\mathrm{m}}$ эnc $\psi$ e the constants which can be determined from the initial conditions.
Puttin. the value of $Q=Q_{0}$ at time $t=0$ in the above equation,
$Q_{l}=\begin{array}{llllll} & \sin \phi & \ldots & \ldots & \ldots & \ldots \\ \ldots & (2)\end{array}$
. ifte.entiating equation (1) with respect to time $t$, we have,
$I=\frac{d Q}{d t}=Q_{m} \omega_{0} \cos \left(\omega_{0} t+\phi\right)$
But at time $\mathbf{t}=\mathbf{0}, \mathrm{I}=\mathbf{0}$.
$\therefore 0=Q_{m} \omega_{0} \cos \phi \Rightarrow \phi=\pi / 2 \quad\left(\because Q_{m}\right.$ and $\omega_{0}$ are not zero $)$
Putting the value of $\phi=\pi / 2$ in equation (2) gives $Q_{m}=Q_{0}$

Putting the value of $\phi=\pi / 2$ and $Q_{m}=Q_{0}$ in equation (1), we have,
$Q=Q_{0} \sin \left(\omega_{0} t+\pi / 2\right)=Q_{0} \cos \omega_{0} t \quad \ldots \quad \ldots \quad \ldots$ (3)
This equation shows that the charge on the capacitor changes in a periodic manner.
Differentiating the above equation with respect to time $t$, we have,
$I=\frac{d Q}{d t}=-Q_{0} \omega_{0} \sin \omega_{0} t$
This equation shows that the current through the inductor also chanaes $\eta$ a $I$ eriodic manner.
At time $t=0$, the charge on the capacitor is maximum and the $c$ rrent through the inductor is zero. The electric field intensity and hence the energy associated .ith the capacitor ( $Q^{2} / 2 C$ ) is maximum. The energy associated with the magr $\therefore$ field of the inductor is zero.

With the passage of time, the charge and hence the $\operatorname{nit}_{\mathrm{L}}$ y y ; ssociated with the capacitor decrease as per the equation (3). At the same tim , the current through the inductor and hence the magnetic field and energy associated with it $\mathbf{L I}^{2} 2$ ) increase as per the equation (4). It can thus be concluded that the energ o the electric field of the capacitor gets converted into the energy of the magnetic fielr' $\mathrm{li}^{\prime} \cdot \mathrm{e}$ Juctor.

At time $t=\pi /\left(2 \omega_{0}\right), Q=0$, and $I$ be nes maximum and the entire energy stored in the electric field gets converted into the ern gy : 'ored in the magnetic field.

At time $t=\pi / \omega_{0}$, the charge on t . canacitor again becomes maximum but with reverse polarity and the current in the ' dl tor ecomes zero. This phenomenon of charge oscillating between the capacitor and indu io' in - periodic manner is known as L-C oscillations.

These oscillations of chary, cults in the emission of electromagnetic radiations which result in the decrease of en ${ }^{\cdots}$ as oclated with the L-C circuit. Such a circuit is known as the tank circuit of the os illa' ur

### 8.11 Power ard Eneiyy associated with L, C and R in an A. C. Circuit

In an A. C. $\quad$. $\quad$ voltage and current continuously change with time. In a series L-C-R circuit, tre 'nsi nta.leous power

$$
\begin{aligned}
& =V_{i} \cdot{ }_{m} \cos \omega t I_{m} \cos (\omega t-\delta) \\
& =V_{m} I_{m} \cos \omega t \cdot \cos (\omega t-\delta)=\frac{V_{m} I_{m}}{2}[\cos \delta+\cos (2 \omega t-\delta)
\end{aligned}
$$

- Keal power, $\mathbf{P}=$ Average value of instantaneous power for the entire cycle

$$
\begin{align*}
& =\frac{V_{m} I_{m}}{2}\left[\frac{1}{T} \int_{0}^{T} \cos \delta d t+\frac{1}{T} \int_{0}^{T} \cos (2 \omega t-\delta) d t\right] \\
& =\frac{V_{m} I_{m}}{2} \cdot \frac{T}{T} \cos \delta \quad\left[\because \frac{1}{T} \int_{0}^{T} \cos (2 \omega t-\delta) d t=0\right. \\
& =\frac{V_{m}}{\sqrt{2}} \cdot \frac{I_{m}}{\sqrt{2}} \cos \delta=V_{\text {r.m.s. }} I_{\text {r.m.s. }} \cos \delta  \tag{1}\\
& \ldots
\end{align*} \ldots
$$

## Special cases:

## (i) Circuit containing only resistor:

$\delta=0 \quad \therefore P=V_{\text {r.m.s. }}$ Ir.m.s.
(ii) Circuit containing only inductor:
$\delta=\pi / 2 \Rightarrow \cos \pi / 2=0 \quad \therefore P=0$
When the current through the inductor increases, the energy from th volt. ye source gets stored in the magnetic field linked with the inductor and is given - on to ae circuit when the current through the inductor decreases. Hence power consur ed $b$ : the circuit is zero. Thus, there is current in an A.C. circuit containing inductor without ons aming any power.
(iii) Circuit containing only capacitor:

Here, $\delta=-\pi / 2 \Rightarrow \cos (-\pi / 2)=0 \quad \therefore P: u$.
The energy consumed in charging the capacitor ... St -ar in the electric field between the plates of the capacitor and is given back to the sir $\mathrm{a}^{2}$ when current in the circuit decreases.
(iv) Series L-C-R circuit:

Here, $\cos \delta=\frac{R}{\left.\sqrt{R^{2}+\left(\omega L-\omega^{1}\right)^{\prime}}\right)}=$
Putting this value of $\cos ^{8}$ i equation (1), we get the value of the power in the series L-C$R$ circuit. It is less than whe oully resistance is present in he circuit. Its value is maximum when $\omega^{2} L C=1$.

Thus, power in A. C. clic.tt containing only inductor or capacitor is zero. The current flowing in such a cirruit is cp"led wattless current.

### 8.12 Tr $\operatorname{lin}^{f} \mathbf{m e}_{1}$

When L . $P=V I$ has to be transmitted over a long distance from the power station to tr ? , $v$ 'hrough the cable having resistance $R, I^{2} R$ amount of power gets lost in the form of he، ' whion is very large. If this power is transmitted at a very high voltage, then the current 1. be less thereby reducing the power loss. Moreover, before supplying this power to the 1 uusehold, its voltage has to be reduced to a proper value.

Both the increase as well as decrease of voltage can be done using a transformer. The transformers, which increase the voltage are called step-up transformers, while those which decrease the voltage are called step-down transformers. There is no appreciable loss of power in the transformers.

## Principle:

Transformers work on the principle of electromagnetic induction.

## Construction:

The figure shows the construction of a step-up transformer and its circuit symbol.

Two coils of wire are wound very close to each other on the rectangular slab of iron. The rectangular slab of iron is made up of several layers of iron plates to minimize the eddy currents and power loss due to them. One of the coils called primary coil, P, is connecteu th dhe A. C. source. The other coil is called the secondary coil, S .

In the step-up transformer shown in the figure, the numb of iurns of the primary coil is less than that of the secondary coil and is made up it thich. copper wire. In the step-down transformer, the arrangement is opposite to that of a : 'ep-ul transformer.

The permeability of the material of the slab is $h$ gh a ? result, all the flux generated by the primary coil remains confined to the core at $\mathrm{d}_{\mathrm{g}}$ is inked to the secondary coil. Therefore, the fluxes, $\Phi_{1}$ and $\Phi_{2}$ linked with the nrimc $v$ ana the secondary coils are proportional to the respective number of turns $N_{1}$ and $\sqrt{2}^{\iota^{\circ}}$ th coils.

$$
\begin{equation*}
\therefore \quad \frac{\text { flux } \Phi_{2} \text { linked with secondary } c_{c}{ }^{\prime}}{\text { flux } \Phi_{1} \text { linked with prima', } c}=\frac{.{\text {. o. of secondary turns } N_{2}}_{i^{\prime}}^{\text {No. of primary turns } N_{1}}}{\text { No }} \tag{1}
\end{equation*}
$$

According to Faraday's law,
the emf induced in $t \geqslant p$ drar, coil, $\varepsilon_{1}=-\frac{d \Phi_{1}}{d t}$ and
the emf inducea in the primary coil, $\varepsilon_{2}=-\frac{d \Phi_{2}}{d t}$
From equ:';on '1, we have, $\quad \Phi_{2}=\frac{N_{2}}{N_{1}} \Phi_{1} \Rightarrow \frac{d \Phi_{2}}{d t}=\frac{N_{2}}{N_{1}} \frac{d \Phi_{1}}{d t}$
$\therefore \varepsilon_{2}=-\frac{I_{2}}{-1} \varepsilon_{1}$ or $\frac{\varepsilon_{2}}{\varepsilon_{1}}=\frac{N_{2}}{N_{1}}=r$

- is known as the transformer ratio. For a step-up transformer, $r>1$.
$\therefore$ the power loss in the transformer is negligible, power input $=$ power output
$\therefore \varepsilon_{2} \mathrm{I}_{2}=\varepsilon_{1} \mathrm{I}_{1} \Rightarrow \frac{\varepsilon_{2}}{\varepsilon_{1}}=\frac{\mathrm{I}_{2}}{\mathrm{I}_{1}}=\frac{\mathrm{N}_{2}}{\mathrm{~N}_{1}}=\mathrm{r}$
The above result is valid for an ideal transformer having no power loss. Actually, some power is lost in the primary coil in the form of heat, some in the magnetization and demagnetization of the iron core and some in the form of eddy currents formed on the surface of the iron core. As a result, the output power is less than the input power.

