This question paper contains **8** printed pages]

Code No. : 23(I) *Roll No.* ....

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STATISTICS

Paper : I

Time Allowed : 3 hours] *[Maximum Marks : 300*

Note : (i) *Answers must be written in* **English.**

(ii) *Number of marks carried by each question are indicated at the end of the question.*

(iii) *Part/Parts of the same question must be answered together and should not be interposed between answers to other questions.*

(iv) ***The answer to each question or part thereof should begin on a fresh page.***

(v) ***Your answers should be precise and coherent.***

(vi) *Attempt **five** questions in all, choosing at most **two** questions from each Section. **Question No. 1 is compulsory.***

P. T. O.

SECTION – I

(Probability)

1. (a) Write down the three basic axioms of probability and show that if the events A and B are such that $A \subset B$ then :

   (i) $P(A) \leq P(B)$, and

   (ii) $P(B-A) = P(B) - P(A)$ 15

   (b) What are the various methods of estimation ? Explain any *two* of them with examples. 15

   (c) State and prove Neyman-Pearson lemma. 15

   (d) Describe the analysis of two-way classified data and give the ANOVA table. 15

2. (a) Define conditional probability and show that it Satisfies axioms of probability. 20

   (b) Let A and B be the respective events that two contracts I and II are completed by certain deadlines and suppose that P (at least one contract is completed by the deadline) = 0.9 and P (both contracts are completed by their deadlines) = 0.5. Calculate the probability P (exactly one contract is completed by its deadline). 20

   (c) Let the events $A_1, A_2, \ldots\ldots, An$ defined on a sample space Ω be such that :

(i) $Ai \cap Aj = \phi$, $i \neq j$,

(ii) $\bigcup_{i=1}^{n} Ai = \Omega$, and

(iii) $P(Ai) > 0$ for all i. Then for any event B on Ω, show that :

$$P(B) = \sum_{i=1}^{n} P(B / Ai) \ . \ P(Ai)$$ 20

3. (a) Define random variable (r. v.) and its distribution function (d. f.). A continuous r. v. X has probability density function (pdf) :

$$f(x) = \frac{2x}{9}, 0 < x < 3$$

$$= 0, \text{ otherwise}.$$

Find :

(i) d. f. of X

(ii) $P(X \leq 2)$

(iii) $P(-1 < x < 1.5)$

(iv) $P(X = 2)$ 20

(b) Calculate the characteristic function (ch. f.) of a r. v. X with pdf :

$$f(x) = \lambda . e^{-\lambda x}, x \geq 0$$

Hence obtain its mean and variance. 20

(c) A bowl contains 8 chips. Three of the chips are red and remaining 5 are blue. Two chips are drawn successively at random and without replacement. What is the probability that the first draw results in a red chip and the second draw results in a blue chip ? 20

4. (a) Given that the joint pdf of r. vs. X and Y is

$$f(x,y) = \frac{x(1+3y^2)}{4}, \begin{array}{l} 0 < x < 2 \\ 0 < y < 1 \end{array}$$

$$= 0, \qquad \text{otherwise}$$

Find :

(i) The marginal pdfs of X and Y,

(ii) The conditional pdf of X given $Y = y$,

(iii) $P\left(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3}\right)$ 20

(b) Explain the weak and strong law of large numbers. 20

(c) Let $u(X)$ be a non-negative function of r. v. X. If $E(u(X))$ exists then for every constant C, show that :

$$P(u(X) \geq C) \leq \frac{E(u(X))}{C}$$

Hence deduce Cheleyshev's inequality. 20

SECTION – II

(Statistical Inference)

5. (a) Define consistent estimator of a parameter. Show that in random sampling from a normal population, the sample mean is a consistent estimator of the population mean. 30

(b) Show that $x(x-1)/n(n-1)$ is an unbiased estimator of θ^2 for the $B(n,\theta)$ distribution. 30

6. (a) How is Cramer-Rao inequality useful in obtaining minimum variance unbiased estimator (MVUE) ? Given the pdf :

$$f(x;\theta) = \frac{e^{-\theta} . \theta^x}{x!}, x = 0, 1, 2, ...; \theta > 0$$

Show that the C-R lower bound is θ / n. 30

(b) Describe sequential probability ratio test (SPRT), operating characteristic (OC) and Average sample number (ASN) functions. Derive an expression for ASN. Also obtain ASN for testing $H_0 : \mu = 10$ against $H_1 : \mu = 20$, where observations follow $N(\mu, 10)$ distribution. (Given $\alpha = 0.04,\ \beta = 0.10$). 30

7. (a) What is meant by confidence interval ? How are confidence intervals obtained for large samples ? Show that for the distribution :

$$f(x;\theta) = \theta \, . \, e^{-\theta x}, \ x \geq 0$$

the confidence limits for θ in large samples with confidence coefficient $\alpha = 0.05$ are given by :

$$\left(1 \pm \frac{1.96}{\sqrt{n}}\right) / \bar{x}$$ 30

(b) Define sufficient and efficient estimator. Given that X_1 and X_2 are two independent observations from a Poisson distribution with mean θ, show that $X_1 + X_2$ is sufficient for θ and $X_1 + 2X_2$ is not sufficient for θ. 30

8. (a) State and prove generalized Neyman Pearson lemma for randomized test : 30

(b) Let θ be a real parameter and let X have pdf :

$$f(x,\theta) = c(\theta) . \exp\{Q(\theta) T(x)\} h(x),$$

where $Q(\theta)$ is strictly monotone. Prove that for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$, there exists a UMPU test φ such that, if $Q(\theta)$ is increasing then :

$$\begin{aligned} \varphi(x) &= 1 \quad \text{if } T(x) > c \\ &= \gamma \quad \text{if } T(x) = c \\ &= 0 \quad \text{if } T(x) < c, \end{aligned}$$

where c and γ are obtained by :

$$E_{\theta_0} \varphi(x) = \alpha.$$ 30

SECTION – III

(Linear Inference & Multivariate Analysis)

9. (a) Define estimable function and show that a parametric function $\underline{l}' \underline{\beta}$ is estimable if and only if $\underline{l}$ can be expressed as a linear combination of rows of X. 30

(b) Define Hotelling's T^2 – statistic. Show that :

$$\lambda^{2/n} = \frac{1}{1 + \frac{T^2}{n-1}}$$

where symbols have their usual meaning. 30

10. (a) For the general linear model $\underline{y} = X\underline{\beta} + \underline{e}$, where X is of order $n \times p \ (p < n)$ matrix, show that there exist no unbiased estimator of $\underline{\beta}$ if rank $(X) < p$. 30

(b) Define multiple correlation coefficient R between x_1 and $(x_2, x_3, \ldots\ldots, x_p)$ and show that $R^2 \geq R_0^2$, where R_0 is multiple correlation coefficient between x_1 and any linear combination of $x_2, x_3, \ldots\ldots\ldots, x_p$. 30

11. (a) Suppose $\underline{X} \cap N_p(\underline{\mu}, \Sigma)$ and let $\underline{X}$ be partitioned as $\underline{X}' = (\underline{X}'_1, \underline{X}'_2)$, with $Cov(\underline{X}_1, \underline{X}_2) = \Sigma_{12}$. Prove that the necessary and sufficient condition for $\underline{X}_1$ and $\underline{X}_2$ to be independent is that $\Sigma_{12} = 0$. 30

(b) Discuss discriminant analysis with reference to the case of two p-variate normal populations. How will you test the goodness of an assigned discriminant function ? 30

12. (a) Discuss the method of fitting an orthogonal polynomials. What are the advantages of using orthogonal polynomials for fitting curvilinear relations ? 30

(b) Define Hotelling's T^2 statistic. Give its applications in testing (i) the significance of mean vector of a multivariate normal distribution; (ii) the equality of mean vectors of two multivariate normal populations. 30

Examrace