14 - PROBABILITY

(Answers at the end of all questions)

(1) Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) $\frac{2}{9}$ (b) $\frac{1}{9}$ (c) $\frac{8}{9}$ (d) $\frac{7}{9}$ [AIEEE 2005]

(2) A random variable X has Poisson distribution with mean 2. Then P (x > 1.5) equals

(a) $\frac{2}{e^2}$ (b) 0 (c) $1 - \frac{3}{e^2}$ (d) $\frac{3}{e^2}$ [AIEEE 2005]

(3) Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are

(a) equally likely and mutually exclusive
(b) equally likely but not independent
(c) independent but not equally likely
(d) mutually exclusive and independent [AIEEE 2005]

(4) Let $x_1, x_2, ..., x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is

(a) 15 (b) 18 (c) 9 (d) 12 [AIEEE 2005]

(5) Probability that A speaks truth is $\frac{4}{5}$ while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is

(a) $\frac{3}{20}$ (b) $\frac{1}{5}$ (c) $\frac{7}{20}$ (d) $\frac{4}{5}$ [AIEEE 2004]

(6) The mean and variance of a random variable x having a binomial distribution are 4 and 2 respectively. Then P (x = 1) is

(a) $\frac{37}{256}$ (b) $\frac{219}{256}$ (c) $\frac{128}{256}$ (d) $\frac{28}{256}$ [AIEEE 2004]

(7) A random variable X has the following probability distribution.

X :	1	2	3	4	5	6	7	8
p (X) :	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = { X is a prime number } and F = { X < 4 }, the probability P (E ∪ F) is

(a) 0.87 (b) 0.77 (c) 0.35 (d) 0.50 [AIEEE 2004]

(8) The events A, B, C are mutually exclusive events such that $P(A) = \frac{3x + 1}{3}$, $P(B) = \frac{1 - x}{4}$ and $P(C) = \frac{1 - 2x}{2}$. The set of possible values of x are in the interval

(a) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (b) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (c) $\left[\frac{1}{3}, \frac{13}{3}\right]$ (d) [0, 1] [AIEEE 2003]

(9) Five horses are in a race. Mr A selects two of the horses at random and bets on them. The probability that Mr A selected the winning horse is

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$ [AIEEE 2003]

(10) The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively. Then, P (X = 1) is

(a) $\frac{1}{32}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$ [AIEEE 2003]

(11) The probabilities of a student getting Ist, IInd and IIIrd division in an examination are respectively $\frac{1}{10}$, $\frac{3}{5}$ and $\frac{1}{4}$. The probability, that a student fails in the examination is

(a) $\frac{197}{200}$ (b) $\frac{27}{100}$ (c) $\frac{83}{100}$ (d) $\frac{33}{200}$ [AIEEE 2002]

(12) A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. If from the selected bag one ball is drawn, then the probability that the ball drawn is red is

(a) $\frac{1}{42}$ (b) $\frac{3}{41}$ (c) $\frac{9}{42}$ (d) $\frac{19}{42}$ [AIEEE 2002]

(13) A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, then the probability that it is rusted or a nail is

(a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{11}{16}$ (d) $\frac{14}{16}$ [AIEEE 2002]

(14) A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that both the socks are of the same colour is

(a) $\frac{9}{108}$ (b) $\frac{18}{108}$ (c) $\frac{36}{108}$ (d) $\frac{48}{108}$ [AIEEE 2002]

(15) A 6-faced fair dice is rolled repeatedly till 1 appears for the first time. The probability that the dice is rolled for even number of times is

(a) $\frac{1}{6}$ (b) $\frac{5}{36}$ (c) $\frac{6}{11}$ (d) $\frac{5}{11}$ [IIT 2005]

(16) Three distinct numbers are chosen randomly from first 100 natural numbers, then the probability that all are divisible by 2 and 3 both is

(a) $\frac{4}{33}$ (b) $\frac{4}{35}$ (c) $\frac{4}{25}$ (d) $\frac{4}{1155}$ [IIT 2004]

(17) Two numbers are chosen from { 1, 2, 3, 4, 5, 6 } one after another without replacement. Find the probability that the smaller of the two is less than 4.

(a) $\frac{4}{5}$ (b) $\frac{1}{15}$ (c) $\frac{1}{5}$ (d) $\frac{14}{15}$ [IIT 2003]

(18) If $P(B) = \frac{3}{4}$, $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$ and $P(A \cap B \cap \bar{C} = \frac{1}{3}$, then $P(B \cap C)$ is

(a) $\frac{1}{12}$ (b) $\frac{3}{4}$ (c) $\frac{5}{12}$ (d) $\frac{23}{36}$ [IIT 2003]

(19) If the integers m and n are chosen at random between 1 and 100, then the probability that the number of the form $7^m + 7^n$ is divisible by 5 equals

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{49}$ [IIT 1999]

(20) The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two and 40% chance of passing in exactly two. Which of the following relations are true?

(a) $p + m + c = \frac{19}{20}$ (b) $p + m + c = \frac{27}{20}$

(c) $pmc = \frac{1}{10}$ (d) $pms = \frac{1}{4}$ [IIT 1999]

(21) If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{3}{16}$ [IIT 1998]

(22) A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{5}$ [IIT 1998]

(23) Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

(a) $\frac{1}{2}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{3}$ [IIT 1998]

(24) If E and F are events with $P(E) \leq P(F)$ and $P(E \cap F) > 0$, then

(a) occurrence of $E \Rightarrow$ occurrence of F
(b) occurrence of $F \Rightarrow$ occurrence of E
(c) non-occurrence of $E \Rightarrow$ non-occurrence of F
(d) none of the above implications holds [IIT 1998]

(25) There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ [IIT 1998]

(26) If $\bar{E}$ and $\bar{F}$ are the complementary events of the events E and F respectively and if $0 < P(F) < 1$, then

(a) $P(E/F) + P(\bar{E}/F) = 1$ (b) $P(E/F) + P(E/\bar{F}) = 1$
(c) $P(\bar{E}/F) + P(E/\bar{F}) = 1$ (d) $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = 1$ [IIT 1998]

(27) If for the three events A, B and C, P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P (exactly one of the events C or A occurs) = p and P (all the three events occur simultaneously) = p^2, where $0 < p < \frac{1}{2}$, then the probability of at least one of the three events A, B and C occurring is

(a) $\frac{3p + 2p^2}{2}$ (b) $\frac{p + p^2}{4}$ (c) $\frac{p + p^2}{2}$ (d) $\frac{3p + 2p^2}{4}$ [IIT 1996]

(28) Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral equals

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$ [IIT 1995]

(29) The probability of India winning a test match against West Indies is 1 / 2. Assuming independence from match to match, the probability that in a 5 match series India's second win occurs at the third test is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ [IIT 1995]

(30) $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then

(a) $P(B/A) = P(B) - P(A)$ (b) $P(A' \cup B') = P(A') + P(B')$
(c) $P(A \cup B') = P(A')P(B')$ (d) $P(A/B) = P(A)$ [IIT 1995]

(31) An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is then,

(a) $\frac{16}{81}$ (b) $\frac{1}{81}$ (c) $\frac{80}{81}$ (d) $\frac{65}{81}$ [IIT 1993]

(32) Let E and F be two independent events. If the probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then P (E) and P (F) respectively are

(a) $\frac{1}{3}, \frac{1}{4}$ (b) $\frac{1}{2}, \frac{1}{6}$ (c) $\frac{1}{6}, \frac{1}{2}$ (d) $\frac{1}{4}, \frac{1}{3}$ [IIT 1993]

(33) India plays two matches each with West Indies and Australia. In any match, the probabilities of India getting points 0, 1 and 2 are 0.45, 0.50 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is

(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250 [IIT 1992]

(34) For any two events A and B in a sample space

(a) $P\left(\frac{A}{B}\right) \geq \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$ is always true

(b) $P(\bar{A}) = P(A) - P(\bar{A})P(B)$ does not hold

(c) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A and B are independent

(d) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A and B are disjoint [IIT 1991]

(35) If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then

(a) E and F are mutually exclusive
(b) E and F^c (the complement of event F) are independent
(c) E^c and F^c are independent (d) $P(E/F) + P(E^c/F) = 1$ [IIT 1989]

(36) One hundred identical coins, each with probability, p, of showing us heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to heads showing on 51 coins, then the value of p is

(a) $\frac{1}{2}$ (b) $\frac{49}{101}$ (c) $\frac{50}{101}$ (d) $\frac{51}{101}$ [IIT 1988]

(37) For two events A and B, $P(A \cup B)$ is
(a) not less than $P(A) + P(B) - 1$ (b) not greater than $P(A) + P(B)$
(c) equal to $P(A) + P(B) - P(A \cup B)$ (d) equal to $P(A) + P(B) + P(A \cup B)$
[IIT 1988]

(38) The probability that at least one of the events A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.4 (e) none of these [IIT 1987]

(39) A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are p, q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(a) $p = q = 1$ (b) $p = q = \frac{1}{2}$ (c) $p = 1, q = 0$

(d) $p = 1, q = \frac{1}{2}$ (e) none of these [IIT 1986]

(40) Three identical dice are rolled. The probability that the same number will appear on each of them is

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) $\frac{3}{28}$ [IIT 1984]

(41) If M and N are two events, the probability that exactly one of them occurs is

(a) $P(M) + P(N) - 2P(M \cap N)$ (b) $P(M) + P(N) - P(M \cap N)$
(c) $P(M^c) + P(N^c) - 2P(M^c \cap N^c)$ (d) $P(M \cap N^c) + P(M^c \cap N)$ [IIT 1984]

(42) Fifteen coupons are numbered 1, 2, ..., 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$ (c) $\left(\frac{3}{5}\right)^7$ (d) none of these [IIT 1983]

(43) If A and B are two events such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\bar{A}/\bar{B})$ is equal to

(a) $1 - P(A/B)$ (b) $1 - P(\bar{A}/B)$
(c) $\frac{1 - P(A \cup B)}{P(\bar{B})}$ (d) $\frac{P(\bar{A})}{P(\bar{B})}$ [IIT 1982]

14 - PROBABILITY

(Answers at the end of all questions)

(44) Two fair dice are tossed. Let X be the event that the first die shows an even number, and Y be the event that the second die shows an odd number. The two events X and Y are

(a) mutually exclusive (b) independent and mutually exclusive
(c) dependent (d) none of these [IIT 1979]

(45) There are n persons ($n \geq 3$), among whom are A and B, who are made to stand in a row in random order. Probability that there are exactly r ($r \leq n - 2$) persons between A and B is

(a) $\frac{n - r}{n(n - 1)}$ (b) $\frac{n - r - 1}{n(n - 1)}$ (c) $\frac{2(n - r - 1)}{n(n - 1)}$ (d) $\frac{2r}{n}$

(46) There are 8 players from which four teams each of two players are formed. What is the probability that two specific players are in one team ?

(a) $\frac{1}{4}$ (b) $\frac{15}{28}$ (c) $\frac{1}{8}$ (d) $\frac{1}{7}$

(47) A natural number is selected from the first 20 natural numbers. The probability that $\frac{x^2 - 15x + 50}{x - 15} < 0$ is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	c	c	b	c	d	b	a	d	a	b	d	c	d	d	d	a	a	a	b

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	a	b	d	b	a,d	a	c	b	c,d	a	a,d	b	a,c	b,c,d	d	a,b,c	c	c	b

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
a,c,d	c	c	d	c	d	b													