

## Modern Physics - Questions

### Q 1.

A single electron orbits around a stationary nucleus of charge  $+Ze$ . Where  $Z$  is a constant and  $e$  is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. **(IIT JEE 1981 – 10 MARKS)**

Find

The value of  $Z$ .

The energy required to excite the electron from the third to the fourth Bohr orbit.

The wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.

The kinetic energy, potential energy, potential energy and the angular momentum of the electron in the first Bohr orbit.

The radius of the first Bohr orbit.

(The ionization energy of hydrogen atom = 13.6 eV, Bohr radius =  $5.3 \times 10^{-11}$  metre, velocity of light =  $3 \times 10^8$  m / sec. Planck's constant =  $6.6 \times 10^{-34}$  joules – sec)

### Q 2.

Hydrogen atom in its ground is excited by means of monochromatic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum ? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV.

**(IIT JEE 1982 – 5 Marks)**

### Q 3.

How many electron, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24 ? **(IIT JEE 1982 – 2 Marks)**

(i) number of electrons =

(ii) number of protons =

(iii) number of neutrons =

### Q 4.

A uranium nucleus (atomic number 92, mass number 238 ) emits an alpha particle and the resultant nucleus emits  $\beta^-$  particle. What are the atomic number and mass number of the final nucleus.

**(IIT JEE 1982 – 2 Marks)**

(i) Atomic number =

(ii) Mass number =

**Q 5.**

Ultraviolet light of wavelengths  $800 \text{ \AA}$  when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy  $1.8 \text{ eV}$  and  $4.0 \text{ eV}$  respectively. Find the value of Planck's constant.

(IIT JEE1983 – 4 Marks)

**Q 6.**

The ionization energy of a hydrogen like Bohr atoms is 4 Rydbergs. (i) What is the wavelength of the radiation emitted when the electrons jumps from the first excited state to the ground state ? (ii) What is the radius of the first orbit for this atom ?

(IIT JEE1984 – 4 Marks)

**Q 7.**

A double ionized Lithium atom is hydrogen-like with atomic number 3.

(IIT JEE1985 – 6 Marks)

(i) Find the wavelength of the radiation required to excite the electron in  $\text{Li}^{++}$  from the first to the third Bohr orbit. (Ionization energy of the hydrogen atom equals  $13.6 \text{ eV}$ .)

(ii) How many spectral lines are observed in the emission spectrum of the above excited system ?

**Q 8.**

There is a stream of neutrons with a kinetic energy of  $0.0327 \text{ eV}$ . If the half life of neutrons is 700 seconds, what fraction of neutrons will decay before they travel a distance of 10 m ?

(IIT JEE 1986 – 6 Marks)

**Q 9.**

A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of  $-1$  volt the anode current  $-I$  (In milli amperes ) is given in terms of plate voltage  $V$  ( in volts) by the algebraic relation.

$$I = 0.125 V - 7.5$$

For grid voltage of  $-3$  volts, the current at anode voltage of 300 volts is 5 milliamperere. Determine the plate resistance ( $r_p$ ), transconductance ( $g_m$ ) and the amplification factor ( $\mu$ ) for the triode.

(IIT JEE 1987 – 7 MARKS)

**Q 10.**

A particle of charge equal to that of an electron,  $-e$ , and mass 208 times the mass of the electron (called a mu -meson) moves in a circular orbit around a nucleus of charge  $+3e$ . (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system.

Derive an expression for the radius of the  $n$ th Bohr orbit.

Find the value, of  $n$  for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

Find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit of the first orbit.

**Q 11.**

A beam of light has three wavelengths  $4144 \text{ \AA}$ ,  $4972 \text{ \AA}$  and  $6216 \text{ \AA}$  with a total intensity of  $3.6 \times 10^{-3} \text{ W m}^{-2}$  equally distributed amongst the three wavelengths. The beam falls normally on an area  $1.0 \text{ cm}^2$  of a clean metallic surface of work function  $2.3 \text{ eV}$ . Assume that there is no loss of light by reflection and that each energetically capable photon ejects an electron. Calculate the number of photoelectrons liberated in two seconds.

(IIT JEE 1989 – 8 MARKS)

**Q 12.**

A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level absorbing monochromatic light of photon energy  $2.7 \text{ eV}$ . Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy  $2.7 \text{ eV}$ , some have energy more and some have less than  $2.7 \text{ eV}$ .

(IIT JEE 1989 – 8 MARKS)

- (i) Find the principal quantum number of the initially excited level B.
- (ii) Find the ionization energy for the gas atoms.
- (iii) Find the maximum and the minimum energies of the emitted photons.

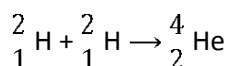
**Q 13.**

Electrons in hydrogen like atom ( $Z = 3$ ) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is  $3.95 \text{ volts}$ . Calculate the work function of the metal and the stopping potential for the photoelectrons ejected by the longer wavelength.

(IIT JEE 1990 – 7 MARKS)

(Rydberg constant =  $1.094 \times 10^7 \text{ m}^{-1}$ )

**Q 14.** It is proposed to use the nuclear fusion reaction



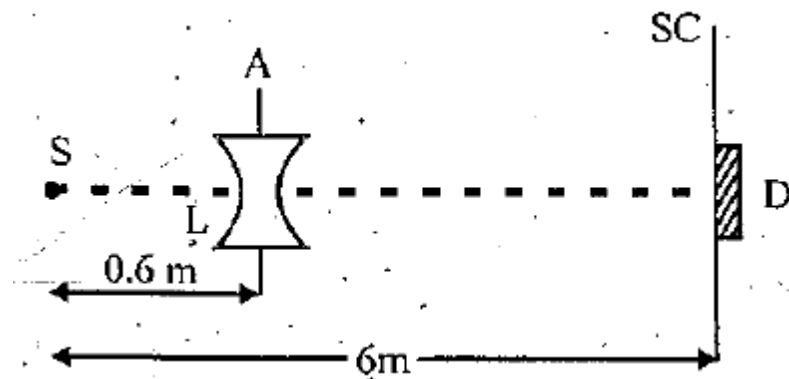
(IIT JEE 1990 – 8 MARKS)

in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day. (The masses of  ${}^2_1\text{H}$  and  ${}^4_2\text{He}$  are 2.0141 atomic mass units and 4.0026 atoms mass units respectively)

**Q 15.**

A monochromatic point source radiating wavelength  $6000 \text{ \AA}$ , with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in fig. A photo emissive detector D of surface area  $0.5 \text{ cm}^2$  is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9.

(IIT JEE 1991 – 2 + 4 + 2 MARKS)



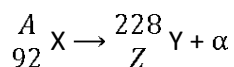
Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

If the work function of the photoemissive surface is 1 eV. Calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

**Q 16.**

A nucleus X, initially at rest, undergoes alpha decay according to the equation,



(IIT JEE 1991 – 4 + 4 MARKS)

Find the values of A and Z in the above process.

The alpha particle produced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 Tesla. Find the energy (In MeV) released during the process and the binding energy of the parent nucleus X.

Given that :  $m(\text{Y}) = 228.03 \text{ u}$ ;  $m({}^1_0\text{n}) = 1.009 \text{ u}$ .

$m({}^4_2\text{He}) = 4.003 \text{ u}$ ;  $m({}^1_1\text{H}) = 1.008 \text{ u}$

**Q 17.**

Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find **(IIT JEE 1992 – 10 MARKS)**

- (a) the energy of the photons causing the photoelectric emission,
- (b) the quantum numbers of the two levels involved in the emission of these photons,
- (c) the change in the angular momentum of the electron in the above transition, and
- (d) the recoil speed of the emitting atom assuming it to be at rest before the transition.

(Ionization potential of hydrogen is 13.6 eV)

**Q 18.**

A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of  $90^\circ$  with respect of its original direction **(IIT JEE 1993 – 9 + 1 MARKS)**

- (i) Find the allowed values of the energy of the neutron and that of the atom after the collision.
- (ii) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation.

[Given : Mass of He atom = 4 × (mass of neutron), Ionization energy of H atom = 13.6 eV]

**Q 19.**

A small quantity of solution containing  $\text{Na}^{24}$  radio nuclide (half life = 15 hour) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume  $1 \text{ cm}^3$  taken. After 5 hour shown an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that radioactive solution mixes uniformly in the blood of the person. (1 curie =  $3.7 \times 10^{10}$  disintegrations per second) **(IIT JEE 1994 – 6 MARKS)**

**Q 20.**

A hydrogen like atom (atomic number Z) is in higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 and 17.0 eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. **(IIT JEE 1994 – 6 MARKS)**

Determine the values of n and Z. ( Ionization energy of H – atom = 13.6 eV)

**Q 21.**

An electron, in a hydrogen-hydrogen-like atom, is in an excited state. It has a total energy of – 3.4 eV. Calculate (i) the kinetic energy and (ii) the de Broglie wavelength of the electron

**(IIT JEE 1996 – 3 MARKS)**

**Q 22.**

At a given instant there are 25% undecayed radio-active nuclei in a sample. After 10 seconds the number of undecayed nuclei reduces to 12.5% Calculate (i) mean-life of the nuclei, and (ii) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number

(IIT JEE 1996 – 3 MARKS)

**Q 23.**

In an ore containing Uranium, the ratio of  $U^{238}$  to  $Pb^{206}$  nuclei is 3. Calculate the age of the ore, assuming that all the lead present in the ore is the final stable product of  $U^{238}$ . Take the half-life of  $U^{238}$  to  $4.5 \times 10^9$  years.

(IIT JEE 1997 C – 5 MARKS)

**Q 24.**

Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if that distance  $d$  between the atoms of the array is  $2\text{\AA}$ . A similar standing wave is again formed if  $d$  is increased to  $2.5\text{\AA}$  but not for any intermediate value of  $d$ . Find the energy of the electrons in electron volts and the least value of  $d$  for which the standing wave of the type described above can form.

(IIT JEE 1997 – 5 MARKS)

**Q 25.**

The element Curium  ${}_{96}^{248}\text{Cm}$  has a mean life of  $10^{13}$  second. Its primary decay modes are spontaneous fission and  $\alpha$  – decay, the former with a probability of 8 % and the latter with a probability of 92 % Each fission releases 200 Me V of energy. The masses involved in  $\alpha$  – decay are as follows.  ${}_{96}^{248}\text{Cm} = 248.072220\text{ u}$ ,  ${}_{94}^{244}\text{Pu} = 244.064100\text{ u}$  and  ${}_{2}^{4}\text{He} = 4.002603\text{ u}$ . Calculate the power output from a sample of  $10^{20}$  Cm atoms. (1 u = 931 Me V/ $c^2$ .)

(IIT JEE 1997 – 5 MARKS)

**Q 26.**

Nuclei of a radioactive element A are being produced at a constant rate  $\alpha$ . The element has a decay constant  $\lambda$ . At time  $t = 0$ , there are  $N_0$  nuclei of the element

(IIT JEE 1998 – 8 MARKS)

(a) Calculate the number  $N$  of nuclei of A at time  $t$ .

(b) If  $\alpha = 2N_0\lambda$ , calculate the number of nuclei of A after one half-life of A, and also the limiting value of  $N$  as  $t \rightarrow \infty$ .

**Q 27.**

Photoelectrons are emitted when 40 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing  $\alpha$  – particles. A maximum energy electron combines with an  $\alpha$ -particle to form a  $\text{He}^+$  ion, emitting a single photon in this process.  $\text{He}^+$  ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV rang, that are likely to be emitted during and after the combination [Take  $h = 4.14 \times 10^{-15}$  eV.s.]

(IIT JEE 1999 – 5 MARKS)

**Q 28.**

A hydrogen- like atom of atom number  $Z$  is in an excited state of quantum number  $2n$ . It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state  $n$ , a photon of energy 40.8 eV is emitted. Find  $n$ ,  $Z$  and the ground state energy (in eV) for this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is – 13. 6 eV.

(IIT JEE 2000 – 6 MARKS)

**Q 29.**

When a beam of 106 eV photon of intensity  $2.0 \text{ W/m}^2$  falls on a platinum surface of area  $1.0 \times 10^{-4} \text{ m}^2$  and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies

(IIT JEE 2000 – 4 MARKS)

**Q 30.**

In a nuclear reaction  $^{235}\text{U}$  undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required.

(IIT JEE 2001 – 5 MARKS)

**Q 31.**

A nucleus at rest undergoes a decay emitting an  $\alpha$  particle of de-Broglie wavelength  $\lambda = 5.76 \times 10^{-15} \text{ m}$ . If the mass of the daughter nucleus is 223. 610 amu and that of the  $\alpha$  particles is 4.002 amu, determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in amu. ( 1 amu = 931. 470 MeV /  $c^2$ )

(IIT JEE 2001 – 5 MARKS)

**Q 32.**

A radioactive nucleus X decays to a nucleus Y with a decay constant  $\lambda_x = 0.1 \text{ s}^{-1}$ . Y further decays to a stable nucleus Z with a decay constant  $\lambda_y = 1/30 \text{ s}^{-1}$ . Initially, there are only X nuclei and their number is  $N_0 = 10^{20}$ . Set up the rate equations for the populations of X, Y and Z. The population of Y nucleus as a function of time is given by  $N_Y(t) = (N_0 \lambda_x (\lambda_x - \lambda_y)) \{ \exp(-\lambda_y t) - \exp(-\lambda_x t) \}$ . Find the time at which  $N_Y$  is maximum and determine the populations X and Z at that instant.

(IIT JEE 2001 – 5 MARKS)

**Q 33.**

A hydrogen – like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels both these values).

(IIT JEE 2002 – 5 MARKS)

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

(Take  $hc = 1240 \text{ eV-nm}$ , ground state energy of hydrogen atom =  $-13.6 \text{ eV}$ )

**Q 34.**

Two metallic plates A and B, each of area  $5 \times 10^{-4} \text{ m}^2$ , are placed parallel to each other at a separation of  $1 \text{ cm}$ . Plate B carries a positive charge of  $33.7 \times 10^{-12} \text{ C}$ . A monochromatic beam of light, with photons of energy  $5 \text{ eV}$  each, starts falling on plate A at  $t = 0$  so that  $10^{16}$  photons fall on it per square meter per second. Assume that one photoelectron is emitted for every  $10^6$  incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value  $2 \text{ eV}$ . Determine

(IIT JEE 2002 – 5 MARKS)

(a) the number of photoelectrons emitted up to  $t = 10 \text{ s}$ ,

(b) the magnitude of the electric field between the plates A and B at  $t = 10 \text{ s}$ , and

(c) the kinetic energy of the most energetic photoelectron emitted at  $t = 10 \text{ s}$  when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B. Take  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N-m}^2$

**Q 35.**

Frequency of a photon emitted due to transition of electron of a certain element from L to K shell is found to be  $4.2 \times 10^{18} \text{ Hz}$ . Using Moseley's law, find the atomic number of the element, given that the Rydberg's constant  $R = 1.1 \times 10^7 \text{ m}^{-1}$ .

(IIT JEE 2003 – 2 MARKS)

**Q 36.**

A radioactive sample emits  $n$   $\beta$ -particles in  $2 \text{ sec}$ . In next  $2 \text{ sec}$  it emits  $0.75 n$   $\beta$ -particle, what is the mean life of the sample ?

(IIT JEE 2003 – 2 MARKS)

**Q 37.**

In a photoelectric experiment set up, photons of energy  $5 \text{ eV}$  falls on the cathode having work function  $3 \text{ eV}$ . (a) If the saturation current is  $i_A = 4 \mu\text{A}$  for intensity  $10^{-5} \text{ W / m}^2$ , then plot a graph between anode potential and current. (b) Also draw a graph for intensity of incident radiation  $2 \times 10^{-5} \text{ W/m}^2$ .

(IIT JEE 2003 – 2 MARKS)



**Q 38.**

A radioactive sample of  $^{238}\text{U}$  decays to Pb through a process for which the half-life is  $4.5 \times 10^9$  years. Find the ratio of number of nuclei of Pb to  $^{238}\text{U}$  after a time of  $1.5 \times 10^9$  years. Given  $(2)^{1/3} = 1.26$ .

(IIT JEE 2004 – 2 MARKS)

**Q 39.**

The photons from the Balmer series in Hydrogen spectrum having wavelength between 450 nm to 700 nm are incident on a metal surface of work function 2 eV. Find the maximum kinetic energy of ejected electron. (Given  $hc = 1242 \text{ eV nm}$ )

(IIT JEE 2004 – 4 MARKS)

**Q 40.**

The potential energy of a particle of mass  $m$  is given by

$$V(x) = \begin{cases} E_0; & 0 \leq x \leq 1 \\ 0; & x > 1 \end{cases}$$

$\lambda_1$  and  $\lambda_2$  are the de-Broglie wavelengths of the particle, when  $0 \leq x \leq 1$  and  $x > 1$  respectively. If the total energy of particle is  $2E_0$ , find  $\lambda_1 / \lambda_2$ .

(IIT JEE 2005 – 2 MARKS)

**Q 41.**

Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is  $(14)^{1/3}$ . Find (a) atomic number of the nucleus. (b) the frequency of  $K_\alpha$  line of the X-ray produced. ( $R = 1.1 \times 10^7 \text{ m}^{-1}$  and  $c = 3 \times 10^8 \text{ m/s}$ )

(IIT JEE 2005 – 4 MARKS)

**Q 42.**

In hydrogen – like atom ( $z = 11$ ),  $n$ th line of Lyman series has wavelength  $\lambda$ . The de- Broglie's wavelength of electron in the level from which it originated is also  $\lambda$ . Find the value of  $n$ ?

(IIT JEE 2006 – 6 Marks)

## Modern Physics - Solutions

### Sol.1.

$$(i) E_2 = -13.6 / 4 Z^2, E_3 = -13.6 / 9 Z^2$$

$$E_3 - E_2 = -13.6 Z^2 (1/9 - 1/4) = +13.6 \times 5 / 36 Z^2$$

$$\text{But } E_3 - E_2 = 47.2 \text{ eV (Given)}$$

$$\therefore 13.6 \times 5 / 36 Z^2 = 47.2 \therefore Z = \sqrt{47.2 \times 36 / 13.6 \times 5} = 5$$

$$(ii) E_4 = -13.6 / 16 Z^2$$

$$\therefore E_4 - E_3 = -13.6 Z^2 [1/16 - 1/9] = -13.6 Z^2 [9 - 16 / 9 \times 16]$$

$$= +13.6 \times 25 \times 7 / 9 \times 16 = 16.53 \text{ eV}$$

$$(iii) E_1 = -13.6 / 1 \times 25 = -340 \text{ eV}$$

$$\therefore E = E_\infty - E_1 = 340 \text{ eV} = 340 \times 1.6 \times 10^{-19} \text{ J} [E_\infty = 0 \text{ eV}]$$

$$\text{But } E = hc / \lambda$$

$$\therefore \lambda = hc / E = 6.6 \times 10^{-34} \times 3 \times 10^8 / 340 \times 10^{-19} \times 1.6 \times 10^{-19} \text{ m}$$

$$(iv) \text{ Total Energy of 1st orbit} = -340 \text{ eV}$$

We know that - (T.E.) = K.E. [in case of electron revolving around nucleus]

$$\text{And } 2\text{T.E.} = \text{P.E.}$$

$$\therefore \text{K.E.} = 340 \text{ eV}; \text{P.E.} = -680 \text{ eV}$$

KEY CONCEPT :

Angular momentum in 1<sup>st</sup> orbit:

According to Bohr's postulate,

$$mvr = nh / 2\pi$$

For  $n = 1$ ,

$$mvr = h / 2\pi = 6.6 \times 10^{-34} / 2\pi = 1.05 \times 10^{-34} \text{ J-s.}$$

(v) Radius of first Bohr orbit

$$r_1 = 5.3 \times 10^{-11} / Z = 5.3 \times 10^{-11} / 5$$

$$= 1.06 \times 10^{-11} \text{ m}$$

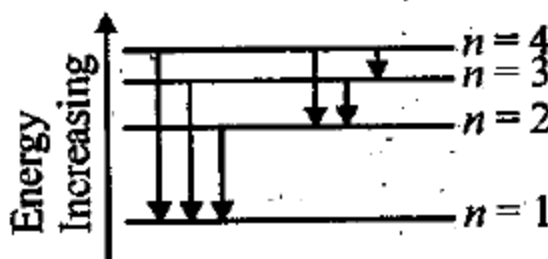
**Sol.2.**

$$E = 12400 / \lambda \text{ (in } \text{\AA}) \text{ eV} = 12400 / 975 = 12.75 \text{ eV} \quad \dots (i)$$

Also

$$13.6 [1 / n_1^2 - 1 / n_2^2] = 12.75 \Rightarrow [1 / 1 - 1 / n_2^2] = 12.75 / 13.6 \Rightarrow n_2 = 4$$

For every possible transition one downward arrow is shown therefore the possibilities are 6.



Note : For longest wavelength, the frequency should be smallest.

This corresponds to the transition from  $n = 4$  to  $n = 3$ , the energy will be  $E_4 = 13.6 / 4^2$ ;  $E_3 = -13.6 / 3^2$

$$\therefore E_4 - E_3 = 13.6 / 4^2 - (-13.6 / 3^2) = 13.6 [1/9 - 1/16]$$

$$= 0.66 \text{ eV} = 0.66 \times 1.6 \times 10^{-19} \text{ J} = 1.056 \times 10^{-19} \text{ J}$$

$$\text{Now, } E = 12400 / \lambda \text{ (in } \text{\AA}) \text{ eV} \therefore \lambda = 18787 \text{ \AA}$$

**Sol.3.**

(i) In a nucleus, number of electrons = 0 ( $\because$  electrons don't reside in the nucleus atom)

(ii) number of protons = 11

(iii) number of neutrons =  $24 - 11 = 13$

**Sol.4.**

(i) Atomic number = 91

(ii) Mass number = 234

**Sol.5.**

$$hc / \lambda_1 - hc / \lambda_0 = \text{K.E.}_1 \quad \dots (i)$$

$$\text{And } hc / \lambda_2 - hc / \lambda_0 = \text{K.E.}_2 \quad \dots (ii)$$

$$\Rightarrow hc / \lambda_1 - hc / \lambda_2 = \text{K.E.}_1 - \text{K.E.}_2$$

$$\Rightarrow hc [\lambda_2 - \lambda_1 / \lambda_1 \lambda_2] = K.E._1 - K.E._2$$

$$\therefore h = (K.E._1 - K.E._2) \lambda_1 \lambda_2 / c (\lambda_2 - \lambda_1)$$

$$= (1.8 - 4) \times 1.6 \times 10^{-19} \times 800 \times 10^{-10} \times 700 \times 10^{-10} / 3 \times 10^8 \times (700 - 800) \times 10^{-10}$$

$$= 6.6 \times 10^{34} \text{ J.s.}$$

#### **Sol.6.**

(i)  $E_n = -I.E. / n^2$  for Bohr's hydrogen atom.

$$\text{Here, } I.E. = 4R \therefore E_n = -4R / n^2$$

$$\therefore E_2 - E_1 = -4R / 2^2 - (-4R / 1^2) = 3R \quad \dots(i)$$

$$E_2 - E_1 = h\nu = hc / \lambda \quad \dots(iii)$$

From (i) and (ii)

$$hc / \lambda = 3R$$

$$\therefore \lambda = hc / 3R = 6.6 \times 10^{-34} \times 3 \times 10^8 / 2.2 \times 10^{-18} \times 3 = 300 \text{ \AA}$$

(ii) The radius of the first orbit

$$\text{Bohr's radius of hydrogen atom} = 5 \times 10^{-11} \text{ m (given)}$$

$$|E_n| = + 0.22 \times 10^{-17} Z^2 = 4R = 4 \times 2.2 \times 10^{-18}$$

$$\therefore Z = 2$$

$$\therefore r_n = r_0 / Z = 5 \times 10^{-11} / 2 = 2.5 \times 10^{-11} \text{ m}$$

#### **Sol.7.**

(i)  $E_n = -13.6 / n^2 Z^2 \text{ eV / atom}$

$$\text{For } Li^{2+}, Z = 3 \therefore E_n = -13.6 \times 9 / n^2 \text{ eV / atom}$$

$$\therefore E_1 = -13.6 \times 9 / 1 \text{ and } E_3 = -13.6 \times 9 / 9 = -13.6$$

$$\Delta E = E_3 - E_1 = -13.6 - (-13.6 \times 9)$$

$$13.6 \times 8 = 108.8 \text{ eV / atom}$$

$$\lambda = 12400 / E \text{ (in eV)} \text{ \AA} = 12400 / 108.8 = 114 \text{ \AA}$$

(ii) The spectral line observed will be three namely  $3 \rightarrow 1$ ,

$3 \rightarrow 2, \rightarrow 1$ .

**Sol.8.**

$$\text{K.E.} = 0.0327 \text{ eV} = 0.327 \times 1.6 \times 10^{-19} \text{ J}$$

$$\frac{1}{2} m_n v_n^2 = 0.0327 \times 1.6 \times 10^{-19}$$

$$\Rightarrow v_n [2 \times 0.0327 \times 1.6 \times 10^{-19} / 1.675 \times 10^{-27}]^{1/2}$$

$$\Rightarrow v_n = 0.25 \times 10^4 \text{ m/s}$$

Time taken by the neutron to travel 10 m will be

$$t = d / v_n = 10 / 0.25 \times 10^4 = 4 \times 10^{-3} \text{ s}$$

Let the number of neutron initially be a.

$$\lambda = 0.693 / t_1 / 2 = 0.693 / 700 \text{ s}^{-1}$$

We know that

$$t = 2.303 / \lambda \log a / a - x$$

$$\Rightarrow 4 \times 10^{-3} / 2.303 \times 0.693 / 700 = 1 = \log_{10} a / a - x$$

$$\Rightarrow \log_{10} a / a - x = 1.72 \times 10^{-6} \Rightarrow a / a - x = 1.000004$$

$$\Rightarrow x / a = 3.96 \times 10^{-6}$$

**Sol.9.**

$$I = 0.125 V - 7.5$$

$$\Rightarrow dl = 0.125 dV \text{ or } dV / dl = 1 / 0.125 = 8$$

We know that plate resistance,  $r_p = dV / dl = 8 \text{ m}\Omega$

The trans conductance,  $g_m = [dl / dV_g] v = \text{constt}$

At  $V_g = -1 \text{ volt}$ ,  $V = 300 \text{ volt}$ , the plate current

$$I = [0.125 \times 300 - 7.5] \text{ mA} = 30 \text{ mA}$$

Also it is given that  $V_g = -3 \text{ V}$ ,  $V = 300 \text{ V}$  and  $I = 5 \text{ mA}$

$$\therefore g_m = [30 - 5 / -1 - (-3)] = 25 / 2 \times 10^{-3} = 12.5 \times 10^{-3} \text{ s}$$

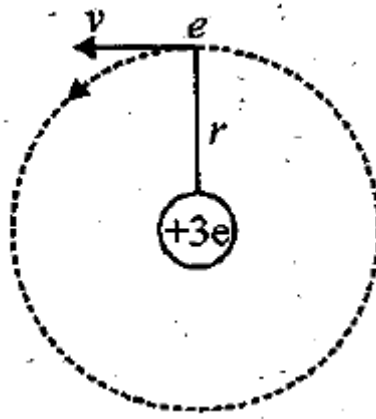
The characteristics are given in the form of parallel lines.

Amplification factor

$$= r_p g_m = 8 \times 10^3 \times 12.5 \times 10^{-3} = 100$$

**Sol.10.**

(i) Let  $m$  be the mass of electron. Then the mass of meson is  $208 m$ . According to Bohr's postulate, the angular momentum of mu-meson should be an integral multiple of  $h/2\pi$ .



$$\therefore (208m) vr = nh / 2\pi$$

$$\therefore v = nh / 2\pi \times 208 mr = nh / 416 \pi mr \quad \dots(i)$$

Note : Since mu-meson is moving in a circular path, therefore, it needs centripetal force which is provided by the electrostatic force between the nucleus and mumesion.

$$\therefore (208m)v^2 / r = 1 / 4\pi\epsilon_0 \times 3e \times e / r^2$$

$$\therefore r = 3e^2 / 4\pi\epsilon_0 \times 208 mv^2$$

Substituting the value of v from (1), we get

$$r = 3e^2 \times 416 \pi mr \times 416 \pi mr / 4 \pi \epsilon_0 \times 208 mn^2 h^2$$

$$\Rightarrow r = n^2 h^2 \epsilon_0 / 624 \pi me^2 \quad \dots(i)$$

(ii) The radius of the first orbit of the hydrogen atom

$$= \epsilon_0 h^2 / \pi me^2 \quad \dots(ii)$$

To find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for hydrogen atom, we equate eq. (i) and (ii)

$$N^2 h^2 / 624 \pi me^2 = \epsilon_0 h^2 / \pi me^2 \Rightarrow n = \sqrt{624} \approx 25$$

$$(iii) 1 / \lambda = 208 R \times Z^2 [1 / n_1^2 - 1 / n_2^2]$$

$$\Rightarrow 1 / \lambda = 208 \times 1.097 \times 10^7 \times 3^2 [1 / 1^2 - 1 / 3^2]$$

$$\Rightarrow \lambda = 5.478 \times 10^{-11} \text{ m}$$

#### **Sol.11.**

$$E_1 = 12400 / 4144 = 2.99 \text{ eV}, E_2 = 12400 / 4972 = 2.49 \text{ eV},$$

$$E_3 = 12400 / 6216 = 1.99 \text{ eV}.$$

$\Rightarrow$  Only first two wavelengths are capable of ejecting photoelectrons.

Energy incident per second

$$= 3.6 / 3 \times 10^{-3} \times 10^{-4} = 1.2 \times 10^{-7} \text{ J/s}$$

$$\therefore n_1 = 1.2 \times 10^{-7} / 2.99 \times 1.6 \times 10^{-19} = 2.5 \times 10^{11}$$

$$n^2 = 1.2 \times 10^{-7} / 2.99 \times 1.6 \times 10^{-19} = 3 \times 10^{11}$$

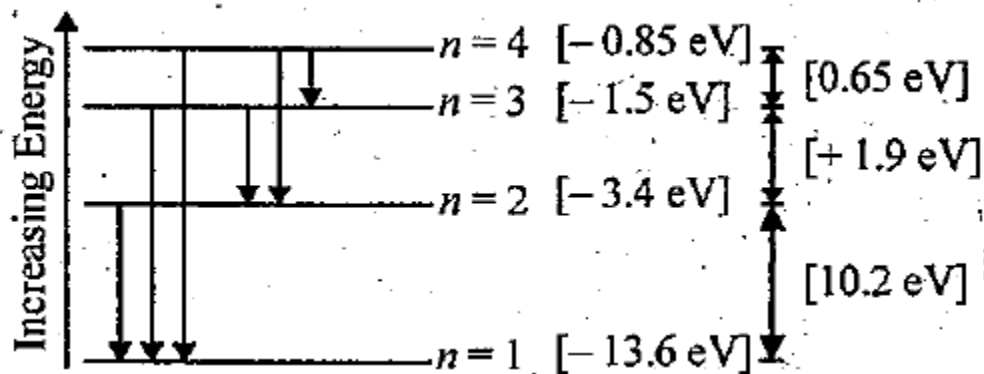
$$\text{Total number of photons} = 2 (n_1 + n_2)$$

$$= 3.01 \times 10^{11} + 2.51 \times 10^{11} = 5.52 \times 10^{11}$$

$$\therefore \text{Total number of photoelectrons ejected in two seconds} = 11 \times 10^{11}$$

**Sol.12.**

(i) The transition of six different photon energies are shown.



Since after absorbing monochromatic light, some of the emitted photons have energy more and some have less than 2.7 eV, this indicates that the excited level B is  $n = 2$ . (This is because if  $n = 3$ ) is the excited level then energy less than 2.7 eV is not possible)

(ii) For hydrogen like atoms we have

$$E_n = -13.6 / n^2 \text{ eV / atom}$$

$$E^4 - E^2 = -13.6 / 16 Z^2 - (-13.6 / 4) Z^2 = 2.7$$

$$\Rightarrow Z^2 \times 13.6 [1/4 - 1/16] = 2.7$$

$$\Rightarrow Z^2 = 2.7 \times 13.6 \times 4 \times 16 / 12 \Rightarrow \text{I.E.} = 13.6 Z^2 (1/1^2 - 1/\infty^2)$$

$$= 13.6 \times 2.7 \times 13.6 \times 4 \times 16 / 12 = 14.46 \text{ eV}$$

(iii) Max. Energy

$$E_4 - E_3 = 13.6 Z^2 (1/4^2 - 1/3^2)$$

$$= 13.6 \times 2.7 \times 13.6 \times 4 \times 16 / 12 \times 15 / 16 = 13.5 \text{ eV}$$

Min. Energy

$$E_4 - E_3 = -13.6 Z^2 (1/4^2 - 1/3^2)$$

$$= 13.6 \times 2.7 \times 13.6 \times 4 \times 16 / 12 \times 7 / 9 \times 16 = 0.7 \text{ eV}$$

**Sol.13.**

For hydrogen like atom energy of the nth orbit is

$$E_n = 13.6 n^2 Z^2 \text{ eV / atom}$$

For transition from  $n = 5$  to  $n = 4$ ,

$$h\nu = 13.6 \times 9 \left[ \frac{1}{16} - \frac{1}{25} \right] = 13.6 \times 9 \times 9 / 16 \times 25 = 2.754 \text{ eV}$$

For transition from  $n = 4$  to  $n = 3$ ,

$$h\nu' = 13.6 \times 9 \left[ \frac{1}{9} - \frac{1}{16} \right] = 13.6 \times 9 \times 7 / 9 \times 16 = 5.95 \text{ eV}$$

For transition  $n = 4$  to  $n = 3$ , the frequency is high and hence wavelength is short.

For photoelectric effect,  $h\nu' - W = eV_0$ , where  $W$  = work function

$$5.95 \times 1.6 \times 10^{-19} - W = 1.6 \times 10^{-19} \times 3.95$$

$$\Rightarrow W = 2 \times 1.6 \times 10^{-19} = 2 \text{ eV}$$

Again applying  $h\nu - W = eV'$

$$\text{We get, } 2.754 \times 1.6 \times 10^{-19} - 2 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19} V'_0$$

$$\Rightarrow V'_0 = 0.754 \text{ V}$$

**Sol.14.**

Energy required per day

$$E = P \times t = 200 \times 10^6 \times 24 \times 60 \times 60$$

$$= 1.728 \times 10^{13} \text{ J}$$

Energy released per fusion reaction

$$= [2 (2.0141) - 4.0026] \times 931.5 \text{ MeV}$$

$$= 23.85 \text{ MeV} = 23.85 \times 10^6 \times 1.6 \times 10^{-19}$$

$$= 38.15 \times 10^{-13} \text{ J}$$

$\therefore$  No. of fusion reactions required

$$= 1.728 \times 10^{13} / 38.15 \times 10^{-13} = 0.045 \times 10^{26}$$

$\therefore$  No. of deuterium atoms required

$$= 2 \times 0.045 \times 10^{26} = 0.09 \times 10^{26}$$

Number of moles of deuterium atoms

$$= 0.09 \times 10^{26} / 6.02 \times 10^{23} = 14.95$$

$\therefore$  Mass in gram of deuterium atoms

$$= 14.95 \times 2 = 29.9 \text{ g}$$



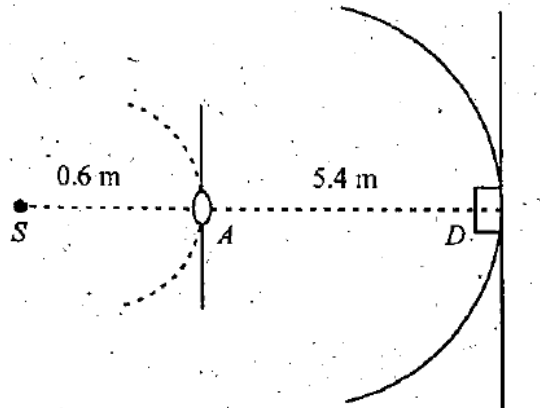
But the efficiency is 25%

Therefore, the actual mass required = 119.6 g

**Sol.15.**

Energy of one photon,  $E = hc / \lambda = (6.6 \times 10^{-34}) (3.0 \times 10^8) / 6000 \times 10^{-10}$

$$= 3.3 \times 10^{-19} \text{ J}$$



Power of the source is 2 W or 2 J/s. Therefore, number of photons emitting per second,

$$N_1 = 2 / 3.3 \times 10^{-19} = 6.06 \times 10^{18} / \text{s}$$

At distance 0.6 m, number of photons incident unit area per unit time:

$$n_2 = n_1 / 4\pi(0.6)^2 = 1.34 \times 10^{18} / \text{m}^2 / \text{s}$$

Area of aperture is,

$$S_1 = \pi / 4 d^2 (0.1)^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$\therefore$  Total number of photons incident per unit time on the aperture,

$$N_3 = n_2 S_1 = (1.34 \times 10^{18}) (7.85 \times 10^{-3}) / \text{s}$$

$$= 1.052 \times 10^{16} / \text{s}$$

The aperture will become new source of light.

Now these photons are further distributed in all directions. Hence, at the location of detector, photons incident per unit area unit time :

$$N_4 = n_3 / 4\pi (5.4 - 0.6)^2 = 1.052 \times 10^{16} / 4\pi (5.4)^2$$

$$= 2.87 \times 10^{13} \text{ s}^{-1} \text{ m}^{-2}$$

This is the photon flux at the centre of the screen. Area of detector is  $0.5 \text{ cm}^2$  or  $0.5 \times 10^{-4} \text{ m}^2$ . Therefore, total number of photons incident on the detector per unit time:

$$n_5 = (0.5 \times 10^{-4}) (2.87 \times 10^{13} \text{ s}^{-1} \text{ m}^{-2}) = 1.435 \times 10^9 \text{ s}^{-1}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time :

$$n_6 = 0.9 n_5 = 1.2915 \times 10^9 \text{ s}^{-1}$$

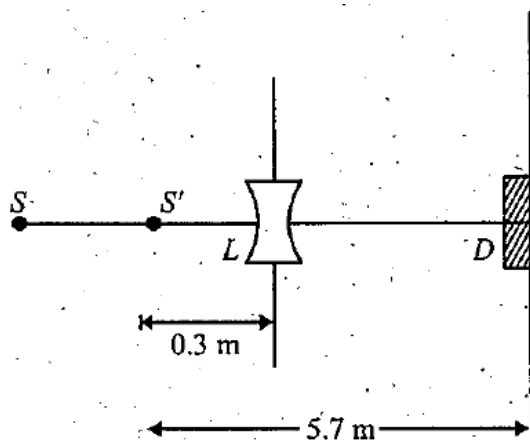
or, photocurrent in the detector :

$$i = (e)n_6 = (1.6 \times 10^{-19})(1.2915 \times 10^9) = 2.07 \times 10^{-10} \text{ A}$$

**(b)** Using the lens formula :

$$1/v - 1/-0.6 = 1/-0.6 \text{ or } v = -0.3 \text{ m}$$

i.e, image of source (say  $S'$ , is formed at 0.3 m from the lens,)



Total number of photons incident per unit on the lens are still  $n_3$  or  $1.052 \times 10^{16}$ s. 80% of it transmits to seconds medium Therefore, at a distance of 5.7 m from  $S'$  number of photons incident per unit are per unit time will be :

$$N_1 = (80 / 100) (1.05 \times 10^{16}) / (4 \pi) (5.7)^2$$

This is the photon flux at the detector.

New value of photocurrent is :

$$i = (2.06 \times 10^{13}) (0.5 \times 10^{-4}) (0.9) (1.6 \times 10^{-19}) \\ = 1.483 \times 10^{-10} \text{ A}$$

**(c)** For stopping potential

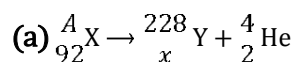
$$hc / \lambda = (E_K)_{\max} + W = eV_0 + W$$

$$\therefore eV_0 = hc / \lambda - W = 3.315 \times 10^{-19} / 1.6 \times 10^{-19} - 1 = 1.07 \text{ eV}$$

$$\therefore V_0 = 1.07 \text{ Volt}$$

Note : The value of stopping potential is not affected by the presence of concave lens as it changes the intensity and not the frequency of photons. The stopping potential depends on the frequency of photons.

#### **Sol.16.**



$$A = 228 + 4 = 232; 92 = Z + 2 \Rightarrow Z = 90$$

**(b)** Let  $v$  be the velocity with which  $\alpha$  - particle is emitted.

Then

$$mv^2/r = qvB \Rightarrow v = \frac{2 \times 1.6 \times 10^{-19} \times 0.11 \times 3}{4.003 \times 10^{-27}}$$

$$\Rightarrow v = 1.59 \times 10^7 \text{ ms}^{-1}$$

Applying law of conservation of linear momentum during  $\alpha$  - decay we get

$$M_y v_y = m_\alpha v_\alpha \quad \dots(1)$$

The total kinetic energy of  $\alpha$  - particle and Y is

$$\begin{aligned} E &= K.E._\alpha + K.E._y = \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_y v_y^2 \\ &= \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_y [m_\alpha v_\alpha / m_y]^2 = \frac{1}{2} m_\alpha v_\alpha^2 + m_\alpha v_\alpha^2 + m_\alpha^2 v_\alpha^2 / 2m_y \\ &= \frac{1}{2} \times 4.033 \times 1.6 \times 10^{-27} \times (1.59 \times 10^7)^2 [1 + 4.003 / 228.03] \\ &= 8.55 \times 10^{-13} \text{ J} \\ &= 5.34 \text{ MeV} \end{aligned}$$

$\therefore$  Mass equivalent of this energy

$$= 5.34 / 931.5 = 0.0057 \text{ a.m.u.}$$

Also,  $m_x + m_\alpha +$  mass equivalent of energy (E)

$$= 228.03 + 4.003 + 0.0057 = 232.03874 \text{ u.}$$

The number of nucleus = 92 protons + 140 neutron.

$\therefore$  Binding energy of nucleus X

$$\begin{aligned} &= [92 \times 1.008 + 140 \times 1.009] - 232.0387 = 1.9571 \text{ u} \\ &= 1.9571 \times 931.5 = 1823 \text{ MeV.} \end{aligned}$$

### **Sol.17.**

**(a)** The energy of photon causing photoelectric emission

= work function of sodium metal + KE of the fastest photoelectron

$$= 1.82 + 0.73 = 2.55 \text{ eV}$$

**(b)** We know that  $E_n = -13.6 / n^2 \text{ eV / atom}$  for hydrogen atom.

Let electron jump from  $n_2$  to  $n_1$  then

$$E_{n_2} - E_{n_1} = -13.6 / n_2^2 - (-13.6 / n_1^2)$$

$$\Rightarrow 2.55 = 13.6 (1/n_1^2 - 1/n_2^2)$$

By hit and trial we get  $n_2 = 4$  and  $n_1 = 2$

$$[\text{angular momentum } mvr = nh / 2\pi]$$

**(c)** Change in angular momentum

$$= n_1 h / 2\pi - n_2 h / 2\pi = h / 2\pi (2 - 4) = h / 2\pi \times (-2) = -h / \pi$$

(d) The momentum of emitted photon can be found by de Broglie relationship

$$\lambda = h / p \Rightarrow p = h / \lambda = hc / c = E / c \quad \therefore p = 2.55 \times 1.6 \times 10^{-19} / 3 \times 10^8$$

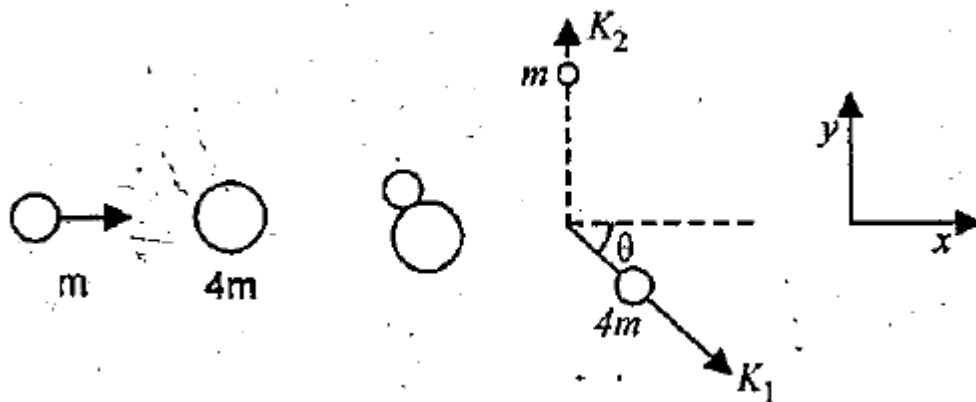
Note : The atom was initially at rest the recoil momentum of the atom will be same as emitted photon (according to the conservation of angular momentum).

Let  $m$  be the mass and  $v$  be the recoil velocity of hydrogen atom then

$$m \times v = 2.55 \times 1.6 \times 10^{-19} / 3 \times 10^8$$

$$\Rightarrow v = 2.55 \times 1.6 \times 10^{-19} / 3 \times 10^8 \times 1.67 \times 10^{-27} = 8.14 \text{ m/s}$$

**Sol.18.**



Applying conservation of linear momentum in horizontal direction

$$(\text{Initial Momentum})_x = (\text{Final Momentum})_x$$

$$(P_i)_x = (P_f)_x$$

$$\Rightarrow \sqrt{2}Km = \sqrt{2}(4m)K_1 \cos \theta \quad \dots(i)$$

Now applying conservation of linear momentum in Y - direction

$$(P_i)_y = (P_f)_y$$

$$0 = \sqrt{2}K_2 m - \sqrt{2}(4m)K_1 \sin \theta$$

$$\Rightarrow \sqrt{2}K_2 m = \sqrt{2}(4m)K_1 \sin \theta \quad \dots(ii)$$

Squaring and adding (i) and (ii)

$$2Km + 2Km_2 m = 2(4m)K_1 + 2(4m)K_1$$

$$K_1 + K_2 = 4K_1 \Rightarrow K = 4K_1 - K_2 \Rightarrow 4K_1 - K_2 = 65 \quad \dots(iii)$$

When collision takes place, the electron gains energy and jumps to higher orbit.

Applying energy conservation

$$K = K_1 + K_2 + \Delta E$$

$$\Rightarrow 65 = K_1 + K_2 + \Delta E \quad \dots(\text{iv})$$

Possible value of  $\Delta E$  for  $\text{He}^+$

Case (1)

$$\Delta E_1 = -13.6 - (-54.4) = 40.8 \text{ eV}$$

$$\Rightarrow K_1 + K_2 = 24.2 \text{ eV from (4)}$$

Solving with (3), we get

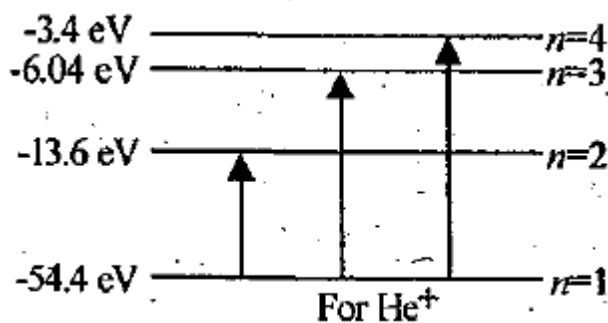
$$K_2 = 6.36 \text{ eV} ; K_1 = 17.84 \text{ eV}$$

Case (2)

$$\Delta E_2 = -6.04 - (-54.4) = 48.36 \text{ eV}$$

$$\Rightarrow K_1 + K_2 = 16.64 \text{ eV from (4)}$$

Solving with (3), we get  $K_2 = 0.312 \text{ eV}$ ;  $K_1 = 16.328 \text{ eV}$



Case (3)

$$\Delta E_3 = -3.4 - (-54.4) = 51.1 \text{ eV}$$

$$\Rightarrow K_1 + K_2 = 14 \text{ eV}$$

Solving with (3), we get

$$K_2 = 15.8 \text{ eV} ; K_1 = -1.8 \text{ eV}$$

But K.E. can never be negative therefore case (3) is not possible.

Therefore, the allowed values of kinetic energies are only that of case (1) and case (2) and electron can jump upto  $n = 3$  only.

(ii) Thus when electron jumps back there are three possibilities

$$n_3 \rightarrow n_1 \text{ or } n_3 \rightarrow n_2 \text{ and } n_2 \rightarrow n_1$$

The frequencies will be

$$\nu_1 = E_3 - E_2 / h ; \nu_2 = E_3 - E_1 / h ; \nu_3 = E_2 - E_1 / h$$

$$\text{i.e., } 1.82 \times 10^{15} \text{ Hz; } 11.67 \times 10^{15} \text{ Hz; } 9.84 \times 10^{15} \text{ Hz}$$

**Sol.19.**

$$t_{1/2} = 15 \text{ hours}$$

Activity initially  $A_0 = 10^{-6}$  Curie (in small quantity of solution of  $^{24}\text{Na}$ ) =  $3.7 \times 10^4$  dps

Observation of blood of volume  $1 \text{ cm}^3$

After 5 hours,  $A = 296$  dpm

The initial activity can be found by the formula

$$t = 2.303 / \lambda \log_{10} A_0 / A \Rightarrow 5 = 2.303 / 0.693 / 15 \times \log_{10} A_0 / 296$$

$$\Rightarrow \log_{10} A_0 / 296 = 5 \times 0.693 / 2.303 \times 15 = 0.3010 / 3 = 0.10033$$

$$\Rightarrow A_0 / 296 = 1.26 \Rightarrow A_0 = 373 \text{ dpm} = 373 / 60 \text{ dps}$$

This is the activity level in  $1 \text{ cm}^3$ . Comparing it with the initial activity level of  $3.7 \times 10^4$  dps we find the volume of blood.

$$V = 3.7 \times 10^4 / 373 / 60 = 5951.7 \text{ cm}^3 = 5.951 \text{ litre}$$

**Sol.20.**

For hydrogen like atoms

$$E_n = -13.6 / n^2 Z^2 \text{ eV / atom}$$

$$\text{Given } E_n - E_2 = 10.2 + 17 = 27.2 \text{ eV} \quad \dots(i)$$

$$E_n - E_3 = 4.24 + 5.95 = 10.2 \text{ eV}$$

$$\therefore E_3 - E_2 = 17$$

$$\text{But } E_3 - E_2 = -13.6 / 9 Z^2 - (-13.6 / 4 Z^2)$$

$$= -13.6 Z^2 [1/9 - 1/4]$$

$$= -13.6 Z^2 [4 - 9 / 36] = 13.6 \times 5 / 36 Z^2$$

$$\therefore 13.6 \times 5 / 36 Z^2 = 17 \Rightarrow Z = 3$$

$$E_n - E_2 = 13.6 n^2 \times 3^2 - [-13.6 / 2^2 \times 3^2]$$

$$= -13.6 [9 / n^2 - 9 / 4] = -13.6 \times 9 [4 - n^2 / 4n^2] \quad \dots(ii)$$

From eq. (i) and (ii),

$$-13.6 \times 9 [4 - n^2 / 4n^2] = 27.2$$

$$\Rightarrow -122.4 (4 - n^2) = 108.8 n^2$$

$$\Rightarrow n^2 = 489.6 / 13.6 = 36 \Rightarrow n = 6$$

**Sol.21.**

$$(i) E_n = -3.4 \text{ eV}$$

The kinetic energy is equal to the magnitude of total energy in this case.

$$\therefore \text{K.E.} = +3.4 \text{ eV}$$

(ii) The de Broglie wavelength of electron

$$\begin{aligned} \Lambda &= h / \sqrt{2mK} = 6.64 \times 10^{-34} / \sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19} \text{ eV}} \\ &= 0.66 \times 10^{-9} \text{ m} \end{aligned}$$

**Sol.22.**

(i) From the given information, it is clear that half life of the radioactive nuclei is 10 sec (since half the amount is consumed in 10 second 12.5% i half of 25% pls. note). Mean life

$$\tau = 1/\lambda = 1/0.693 / t_{1/2} = t_{1/2} / 0.693 = 10 / 0.693 = 14.43 \text{ sec}$$

$$(ii) N = N_0 e^{\lambda t}$$

$$N/N_0 = 6.25 / 100$$

$$\Lambda = 0.0693 \text{ s}^{-1}$$

$$6.25 / 100 = e^{-0.0693t}$$

$$e^{+0.693t} = 100 / 6.25 = 16$$

$$0.0693t = \ln 16 = 2.773$$

$$\text{Or } t = 2.733 / 0.0693 = 40 \text{ sec.}$$

**Sol.23.**

Number of atoms of  $^{238}\text{U}$  initially / Number of atoms of  $^{238}\text{U}$  finally =  $4 / 3 = a / (a - x)$

[ $\because$  Initially one part lead is present with three parts Uranium]

$$\therefore t = 2.303 / \lambda \log \alpha / (\alpha - x) = 2.303 \times 4.5 \times 10^9 / 0.693 \log 4/3$$

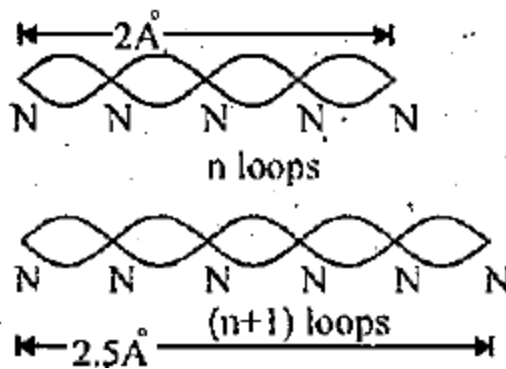
$$= 1.868 \times 10^9 \text{ years.}$$

**Sol.24.**

As nodes are formed at each of the atomic sites, hence

$$2\text{\AA} = n(\lambda / 2) \quad \dots(1)$$

[ $\because$  Distance between successive nodes =  $\lambda / 2$ ]



and  $2.5\text{ Å} = (n + 1) \lambda / 2$

$\therefore 2.5 / 2 = n + 1 / n, 5 / 4 = n + 1 / n$  or  $n = 4$

Hence, from equation (1),

$2\text{ Å} = 4 \lambda / 2$  i.e.,  $\lambda = 1\text{ Å}$

Now, de Broglie wavelength is given by

$\lambda = h / \sqrt{2mK}$  or  $K = h^2 / \lambda^2 2m$

$\therefore K = (6.63 \times 10^{-34})^2 / (1 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \text{ eV}$

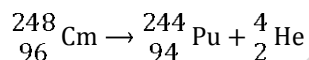
$= (6.63)^2 / 8 \times 9.1 \times 1.6 \times 10^2 \text{ eV} = 151 \text{ eV}$

d will be minimum, when

$n = 1, d_{\min} = \lambda / 2 = 1\text{ Å} / 2 = 0.5\text{ Å}$

### Sol 25.

The reaction involved in  $\alpha$  - decay is



Mass defect

$\Delta m = \text{Mass of } {}_{96}^{248}\text{Cm} - \text{Mass of } {}_{94}^{244}\text{Pu} - \text{Mass of } {}_2^4\text{He}$

$= (248.072220 - 244.064100 - 4.002603)\text{u}$

$= 0.005517 \text{ u}$

Therefore, energy released in  $\alpha$  - decay will be

$E_{\alpha} = (0.005517 \times 931) \text{ MeV} = 5.136 \text{ MeV}$

Similarly,  $E_{\text{fission}} = 200 \text{ MeV}$  (given)

Mean life is given as  $t_{\text{mean}} = 10^{13} \text{ s} = 1 / \lambda$

$\therefore$  Disintegration constant  $\lambda = 10^{-13} \text{ s}^{-1}$

Rate of decay at the moment when number of nuclei are  $10^{20}$  is

$dN / dt = \lambda N = (10^{-13}) (10^{20}) = 10^7 \text{ dps}$



Of these disintegrations, 8% are in fission and 92% are in  $\alpha$  - decay.

Therefore, energy released per second

$$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7) \times 5.136 \text{ MeV}$$

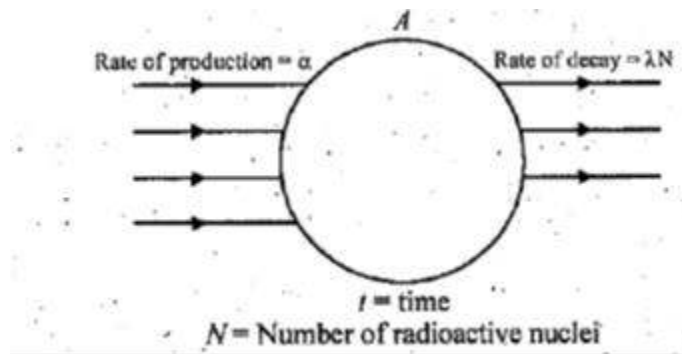
$$= 2.074 \times 10^8 \text{ MeV}$$

$\therefore$  Power output (in watt) = Energy released per second (J/s)

$$= (2.074 \times 10^8) (1.6 \times 10^{-13})$$

$$\therefore \text{Power output} = 3.32 \times 10^{-5} \text{ watt.}$$

### **Sol 26.**



(a) Let at time 't' number of radioactive are N.

Net rate of formation of nuclei of A.

$$dN / dt = \alpha - \lambda N \text{ or } dN / \alpha - \lambda N = dt$$

$$\text{or } \int_{N_0}^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving this equation, we get

$$N = 1 / \lambda [\alpha - (\alpha - \lambda N_0)e^{-\lambda t}] \quad \dots(1)$$

(b) Substituting  $\alpha = 2\lambda N_0$  and

$$t = t_{1/2} = 1 \ln(2) / \lambda \text{ in equation (1),}$$

$$\text{we get, } N = 3/2 N_0$$

(ii) Substituting  $\alpha = 2\lambda N_0$  and  $t \rightarrow \infty$  in equation (1), we get

$$N = \alpha / \lambda = 2 N_0$$

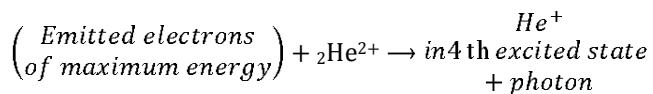
### **Sol 27.**

The energy of the incident photon is

$$E_1 = hc / \lambda = (4.14 \times 10^{-15} \text{ eVs}) (3 \times 10^8 \text{ m/s}) / (400 \times 10^{-9} \text{ m}) = 3.1 \text{ eV}$$

The maximum kinetic energy of the electrons is  $E_{\max} = E_1 - W = 3.1 \text{ eV} - 1.9 \text{ eV} = 1.2 \text{ eV}$

It is given that,



The fourth excited state implies that the electron enters the  $n = 5$  state.

In this state its energy is

$$E_5 = - (13.6 \text{ eV}) Z^2 / n^2 = - (13.6 \text{ eV}) (2)^2 / 5^2 \\ = - 2.18 \text{ eV}$$

This energy of the emitted photon in the above combination reaction is

$$E = E_{\max} + (-E_5) = 1.2 \text{ eV} + 2.18 \text{ eV} = 2.4 \text{ eV}$$

Note : After the recombination reaction, the electron may undergo transition from a higher level to a lower level thereby emitting photons.

The energies in the electronic levels of  $\text{He}^+$  are

$$E_4 = (-13.6 \text{ eV}) (2^2) / 4^2 = -3.4 \text{ eV}$$

$$E_3 = (-13.6 \text{ eV}) (2^2) / 3^2 = -6.04 \text{ eV}$$

$$E_2 = (-13.6 \text{ eV}) (2^2) / 2^2 = -13.6 \text{ eV}$$

The possible transitions are

$$n = 5 \rightarrow n = 4$$

$$\Delta E = E_5 - E_4 = [-2.18 - (-3.4)] \text{ eV} = 1.28 \text{ eV}$$

$$n = 5 \rightarrow n = 3$$

$$\Delta E = E_5 - E_3 = [-2.18 - (-6.04)] \text{ eV} = 3.84 \text{ eV}$$

$$n = 5 \rightarrow n = 2$$

$$\Delta E = E_5 - E_2 = [-2.18 - (-13.6)] \text{ eV} = 11.4 \text{ eV}$$

$$n = 4 \rightarrow n = 3$$

$$\Delta E = E_4 - E_3 = [-3.4 - (-6.04)] \text{ eV} = 2.64 \text{ eV}$$

### **Sol 28.**

Energy for an orbit of hydrogen like atoms is

$$E_n = -13.6 Z^2 / n^2$$

For transition from  $2n$  orbit to  $1$  orbit

$$\text{Maximum energy} = 13.6 Z^2 (1/1 - 1/(2n)^2)$$

$$\Rightarrow 204 = 13.6 Z^2 (1/1 - 1/4n^2) \dots (i)$$

Also for transition  $2n \rightarrow n$ .

$$40.8 = 13.6 Z^2 (1/n^2 - 1/4n^2) \Rightarrow 40.8 = 13.6 Z^2 (3/4n^2)$$

$$\Rightarrow 40.8 = 40.8 Z^2 / 4n^2 = Z^2 \text{ or } 2n = Z \dots (ii)$$

From (i) and (ii)

$$204 = 13.6 Z^2 (1 - 1/Z^2) = 13.6 Z^2 - 13.6$$

$$13.6 Z^2 = 204 + 13.6 = 217.6$$

$$Z^2 = 217.6 / 13.6 = 16, Z = 4, n = Z / 2 = 4 / 2 = 2$$

$$\text{orbit no.} = 2n = 4$$

For minimum energy = Transition from 4 to 3.

$$E = 13.6 \times 4^2 (1/3^2 - 1/4^2) = 13.6 \times 4^2 (7/9 \times 16)$$

$$= 10.5 \text{ eV.}$$

$$\text{Hence } n = 2, Z = 4, E_{\min} = 10.5 \text{ eV}$$

### **Sol 29.**

No. of photons /sec

$$= \text{Energy incident on platinum surface per second} / \text{Energy of on photon}$$

No. of photon incident per second

$$= 2 \times 10 \times 10^{-4} / 10.6 \times 1.6 \times 10^{-19} = 1.18 \times 10^{14}$$

As 0.53% of incident photon can eject photoelectrons

$\therefore$  No. of photoelectrons ejected per second

$$= 1.18 \times 10^{14} \times 0.53 / 100 = 6.25 \times 10^{11}$$

Minimum energy = 0 eV,

$$\text{Maximum energy} = (10.6 - 5.6) \text{ eV} = 5 \text{ eV}$$

### **Sol 30.**

The formula for  $\eta$  of power will be

$$\eta = P_{\text{out}} / P_{\text{in}}$$

$$\therefore P_{\text{in}} = P_{\text{out}} / \eta = 1000 \times 10^6 / 0.1 = 10^{10} \text{ W}$$

Energy required for this power is given by

$$E = p \times t$$

$$= 10^{10} \times 86,400 \times 365 \times 10$$

$$= 3.1536 \times 10^{18} \text{ J}$$

$200 \times 1.6 \times 10^{-13} \text{ J}$  of energy is released by 1 fission

$\therefore 3.1536 \times 10^{18} \text{ J}$  of energy is released by

$$3.1536 \times 10^{18} / 200 \times 1.6 \times 10^{-13} \text{ fission}$$

$$= 0.9855 \times 10^{29} \text{ fission}$$

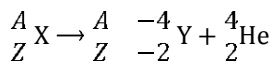
$$= 0.985 \times 10^{29} \text{ of } U^{235} \text{ atoms.}$$

$6.023 \times 10^{23}$  atoms of Uranium has

$$235 \times 0.9855 \times 10^{29} / 6.023 \times 10^{23} \text{ g} = 38451 \text{ kg}$$

### **Sol 31.**

Let the reaction be



Here,  $m_y = 223.61 \text{ amu}$  and  $m_\alpha = 4.002 \text{ amu}$

We know that

$$\lambda = h / mv \Rightarrow m^2 v^2 = h^2 / \lambda^2 = p^2$$

$$\Rightarrow \text{But E.K.} = p^2 / 2m. \text{ Therefore K.E.} = h^2 / 2m\lambda^2 \dots(i)$$

Applying eq. (i) for Y and  $\alpha$ , we get

$$\text{K.E.}_\alpha = (6.6 \times 10^{-34})^2 / 2 \times 4.002 \times 1.67 \times 10^{-27} \times 5.76 \times 10^{-15} \times 5.76 \times 10^{-15}$$

$$= 0.0982243 \times 10^{-11} = 0.982 \times 10^{-12} \text{ J}$$

$$\text{Similarly (E.K.)}_y = 0.0178 \times 10^{-12} \text{ J}$$

$$\text{Total energy} = 10^{-12}$$

We know that  $E = \Delta mc^2$

$$\therefore \Delta m = E / c^2 = 10^{-12} / (3 \times 10^8)^2 \text{ kg}$$

$$1.65 \times 10^{-27} \text{ kg} = 1 \text{ amu}$$

$$\therefore 10^{-12} / (3 \times 10^8)^2 \text{ kg} = 10^{-12} \text{ amu} / 1.67 \times 10^{-27} \times (3 \times 10^8)^2$$

$$= 10^{-12} \text{ amu} / 1.67 \times 9 \times 10^{-27} \times 10^{16} = 0.00665 \text{ amu}$$

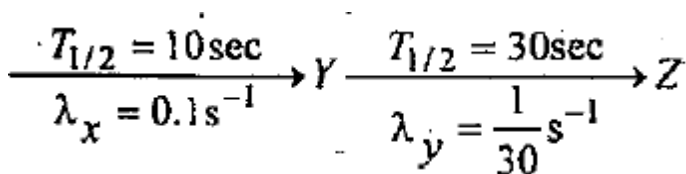
The mass of the parent nucleus X will be

$$M_x = m_y + m_\alpha + \Delta m$$

$$= 223.61 + 4.002 + 0.00665 = 227.62 \text{ amu}$$

**Q 32.**

X



The rate of equation for the population of X, Y and Z will be

$$dN_x / dt = -\lambda_x N_x \quad \dots(i)$$

$$dN_y / dt = -\lambda_y N_y + \lambda_x N_x \quad \dots(ii)$$

$$dN_z / dt = \lambda_y N_y \quad \dots(iii)$$

⇒ On integration, we get

$$N_x = N_0 / \lambda_x - \lambda_y [e^{-\lambda_y t} - e^{-\lambda_x t}]$$

To determine the maximum  $N_y$ , we find

$$dN_y / dt = 0$$

From (ii)

$$-\lambda_y N_y + \lambda_x N_x = 0$$

$$\Rightarrow \lambda_x N_x = \lambda_y N_y \quad \dots(v)$$

$$\Rightarrow \lambda_x (N_0 e^{-\lambda_x t}) = \lambda_y [\lambda_x N_0 / \lambda_x - \lambda_y (e^{-\lambda_y t} - e^{-\lambda_x t})]$$

$$\Rightarrow \lambda_x - \lambda_y / \lambda_y = e^{-\lambda_y t} - e^{-\lambda_x t} / e^{-\lambda_x t} \Rightarrow \lambda_x / \lambda_y = e^{(\lambda_x / \lambda_y - 1)t}$$

$$\Rightarrow \log_e \lambda_x / \lambda_y = (\lambda_x / \lambda_y - 1)t$$

$$\Rightarrow t = \log_e (\lambda_x / \lambda_y) / \lambda_x - \lambda_y = \log_e [0.1 / (1/30)] / 0.1 - 1/30 = 15 \log_e 3$$

$$\therefore N_x = N_0 e^{-0.1(15 \log_e 3)} = N_0 e^{\log_e (3^{-1.5})}$$

$$\Rightarrow N_x = N_0 3^{-1.5} = 10^{20} / 3 \sqrt{3}$$

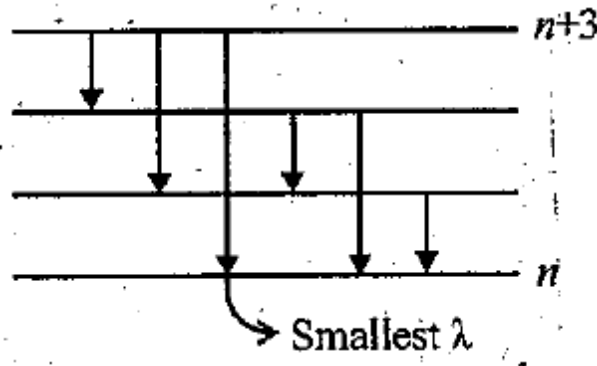
$$\text{Since, } dN_y / dt = 0 \text{ at } t = 15 \log_e 3, \quad \therefore N_y = \lambda_x N_x / \lambda_y = 10^{20} / \sqrt{3}$$

$$\text{And } N_z = N_0 - N_x - N_y$$

$$= 10^{20} - (10^{20} / 3\sqrt{3}) - 10^{20} / \sqrt{3} = 10^{20} (3\sqrt{3} - 4 / 3\sqrt{3})$$

**Sol 33.**

(a) If  $x$  is the difference in quantum number of the states than  $^{x+1}C_2 = 6 \Rightarrow x = 3$



Now, we have  $-z^2 (13.6\text{eV}) / n^2 = -0.85\text{eV}$  ... (i)

And  $-z^2 (13.6\text{ eV}) / (n + 3)^2 = -0.544\text{ eV}$  ... (ii)

Solving (i) and (ii) we get  $n = 12$  and  $z = 3$

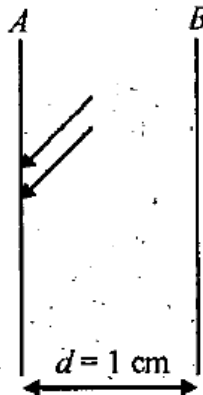
(b) Smallest wavelength  $\lambda$  is given by

$$hc / \lambda = (0.85 - 0.544)\text{ eV}$$

Solving, we get  $\lambda = 4052\text{ nm}$ .

#### Sol 34.

(a) Number of electron falling on the metal plate A =  $10^{16} \times (5 \times 10^{-4})$



$\therefore$  Number of photoelectrons emitted from metal plate A upto 10 seconds is

$$N_e = (5 \times 10^4) \times 10^{16} / 10^6 \times 10 = 5 \times 10^7$$

(b) Charge on plate B at  $t = 10\text{ sec}$

$$Q_b = 33.7 \times 10^{-12} - 5 \times 10^7 \times 1.6 \times 10^{-19} = 25.7 \times 10^{-12}\text{ C}$$

$$\text{Also } Q_a = 8 \times 10^{-12}\text{ C}$$

$$E = \sigma_B / 2\epsilon_0 - \sigma_A / 2\epsilon_0 = 1 / 2A \epsilon_0 (Q_B - Q_A)$$

$$= 17.7 \times 10^{-12} / 5 \times 10^{-4} \times 8.85 \times 10^{-12} = 2000\text{ N/C}$$

(c) K.E. of most energetic particles

$$= (h\nu - \phi) + e (Ed) = 23 \text{ eV}$$

Note :  $(h\nu - \phi)$  is energy of photoelectrons due to light  $e (Ed)$  is the energy of photoelectrons due to work done by photoelectrons between the plates.

### Sol 35.

According to Bohr's model, the energy released during transition from  $n_2$  to  $n_1$  is given by

$$\Delta E = h\nu = Rhc (Z - b)^2 [1 / n_1^2 - 1 / n_2^2]$$

For transition from L shell to K shell

$$b = 1, n_2 = 2, n_1 = 1$$

$$\therefore (Z - 1)^2 Rhc [1/1 - 1/4] = h\nu$$

On putting the value of  $R = 1.1 \times 10^7 \text{ m}^{-1}$  (given),

$c = 3 \times 10^8 \text{ m/s}$ , we get

$$Z = 42$$

### Sol 36.

$$\lambda = \log_e A_0 / A / t = 1 / 2 \log_e n / 0.75 n$$

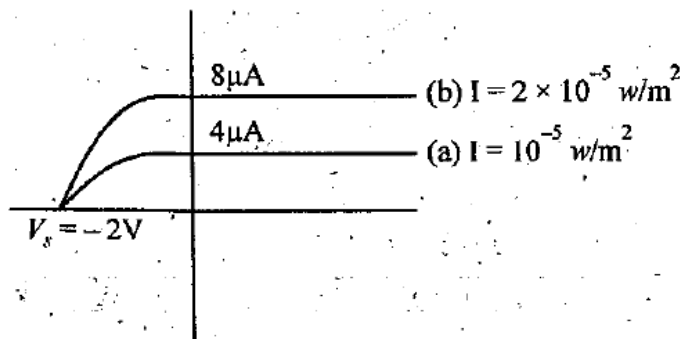
$$\Rightarrow \text{Mean Life} = 1 / \lambda = 2 / \log_e 4 / 3$$

### Sol 37.

$$(a) eV_0 = h\nu - h\nu_0 = 5 - 3 = 2 \text{ eV}$$

$$\therefore V_0 = 2 \text{ volt.}$$

(b) Note : When the intensity is doubled, the saturation current is also doubled,



### Sol 38.

$a$  = Initial Uranium atom

$(a - x)$  = Uranium atoms left

$$(a - x) = a (1/2)^n$$

$$\text{and } n = t / t_{1/2} = 1.5 \times 10^9 / 4.5 \times 10^8 = 3$$

$$\therefore a - x = a (1/2)^{1/3}$$

$$\Rightarrow a / a - x = 1 / (1/2)^{1/3} = 2^{1/3} / 1 = 1.26$$

$$\Rightarrow x / a - x = 1.26 - 1 = 0.26$$

### **Sol 39.**

KEY CONCEPT:

The wavelength  $\lambda$ , of photon for different lines of Balmer series is given by

$$hc / \lambda = 13.6 [1/2^2 - 1/n^2] \text{ eV, where } n = 3, 4, 5$$

Using above relation, we get the value of  $\lambda = 657 \text{ nm}$ ,  $487 \text{ nm}$  between  $450 \text{ nm}$  and  $700 \text{ nm}$ . Since  $487 \text{ nm}$ , is smaller than  $657 \text{ nm}$  electron of max. E.K. will be emitted for photon corresponding to wavelength  $487 \text{ nm}$  with

$$(\text{K.E.}) = hc / \lambda - W = (1242 / 487 - 2) = 0.55 \text{ eV}$$

### **Sol 40.**

The de Broglie wave length is given by

$$\lambda = h / mv \Rightarrow \lambda = h / \sqrt{2mK}$$

Case (i)  $0 \leq x \leq 1$

For this, potential energy is  $E_0$  (given)

Total energy =  $2E_0$  (given)

$$\therefore \text{Kinetic energy} = 2E_0 - E_0 = E_0$$

$$\lambda_1 = h / \sqrt{2mE_0} \quad \dots(i)$$

Case (ii)  $x > 1$

For this, potential energy = 0 (given)

Here also total energy =  $2E_0$  (given)

$$\therefore \text{Kinetic energy} = 2E_0$$

$$\therefore \lambda_2 = h / \sqrt{2m(2E_0)} \quad \dots(ii)$$

Dividing (i) and (ii)

$$\lambda_1 / \lambda_2 = \sqrt{2E_0} / E_0 \Rightarrow \lambda_1 / \lambda_2 = \sqrt{2}$$



**Sol 41.**

(a) KEY CONCEPT : We know that radius of nucleus is given by formula

$r = r_0 A^{1/3}$  where  $r_0 = \text{const}$ , and  $A = \text{mass number}$ .

For the nucleus  $r_1 = r_0 4^{1/3}$

For unknown nucleus  $r_2 = r_0 (A)^{1/3}$

$$\therefore r_2 / r_1 = (A / 4)^{1/3}, (14)^{1/3} = (A / 4)^{1/3} \Rightarrow A = 56$$

$$\therefore \text{No of proton} = A - \text{no. of neutrons} = 56 - 30 = 26$$

(b) We know that  $v = Rc (Z - b)^2 [1 / n_1^2 - 1 / n_2^2]$

Here,  $R = 1.1 \times 10^7$ ,  $c = 3 \times 10^8$ ,  $Z = 26$

$b = 1$  (for  $K_\alpha$ ),  $n_1 = 1$ ,  $n_2 = 2$

$$\therefore v = 1.1 \times 10^7 \times 3 \times 10^8 [26 - 1]^2 [1 / 1^2 - 1 / 4]$$

$$= 3.3 \times 10^{15} \times 25 \times 25 \times \frac{3}{4} = 1.546 \times 10^{18} \text{ Hz}$$

**Sol 42.**

Note :  $n$ th line of Lyman series means electron jumping from  $(n + 1)$ th orbit to 1<sup>st</sup> orbit.

For an electron to revolve in  $(n + 1)$ th orbit.

$$2\pi r = (n + 1) \lambda$$

$$\Rightarrow \lambda = 2\pi / (n + 1) \times r = 2\pi / (n + 1) [0.529 \times 10^{-10}] (n + 1)^2 / Z$$

$$\Rightarrow 1 / \lambda = Z / 2\pi [0.529 \times 10^{-10}] (n + 1) \quad \dots(i)$$

Also we know that when electron jumps from  $(n + 1)$ th orbit to 1<sup>st</sup> orbit

$$1 / \lambda = RZ^2 [1 / 1^2 - 1 / (n + 1)^2] = 1.09 \times 10^7 Z^2 [1 - 1 / (n + 1)^2]$$

From (i) and (ii)

$$Z / 2\pi (0.529 \times 10^{-10}) (n + 1) = 1.09 \times 10^7 Z^2 [1 - 1 / (n + 1)^2]$$

On solving, we get  $n = 24$

Examrace